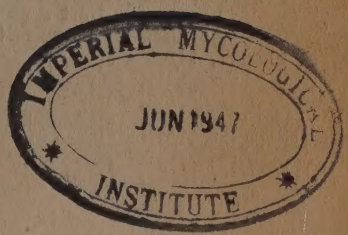
COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1945



VOLUME XLVII

№ 1

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
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Paraissant tous les dix jours

1945

VOLUME XLVII, № 1

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Édition de l'Académie des Sciences de l'URSS

Moscou 1945

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

MATHEMATICS

NORMED RINGS GENERATED BY THE GENERALIZED OPERATION OF TRANSLATION

By B. LEWITAN

(Communicated by A.N. Kolmogoroff, Member of the Academy, 27. V. 1944)

Gelfand and Raikoff ([1]) have successively applied the theory of normed rings developed by Gelfand ([2]) to the investigation of commutative topological groups. An important rôle in their method is played by the operation of multiplication in the ring (the composition, «Faltung»). In the present note the same methods will be used to show that to every commutative operation of generalized translation * a certain normed ring is related. In the discrete case, where the operator of generalized transformation is defined by means of a cubic matrix, Haar's ([3]) results follow from ours.

1. Let Z be a locally compact space with the second enumerability axiom, and $m(\Delta)$ a measure in it, which is nowhere trivial (*i. e.* for every open set U: $m(U) > 0$). By $L_p\,(p \geqslant 1)$ we denote the aggregate of all numerical functions defined on Z for which

$$\int_Z |f(t)|^p\, dm(t) < \infty$$

In the sequel we shall be interested in the cases $p=1$ and $p=2$. The integrals over the whole Z we shall write without the lower index.

Suppose that in the linear function spaces $L_p\,(p \geqslant 1)$ a family of operators T^s is defined that depend on the element $s \in Z$. Thus, to every function $f(t)$ of the considered classes a function of two variables $K(s,\ t) = T^s_t f(t)$ is correlated. By $\tilde{T}^s$ we denote the aggregate of adjoined operators, determined by the relation

$$\int T^s_t f(t)\, g(t)\, dm(t) = \int f(t)\, \tilde{T}^s_t g(t)\, dm(t)$$

It is assumed that the integrals exist for almost all s.

2. Let the following conditions be satisfied, which in the sequel shall be briefly termed the conditions of generalized translation.

I. There exists such an infinite sequence of functions $x_n(t) \in L_{1,2}$ uniformly bounded by the norm in L_1, $\int |x_n(t)|\, dm(t) < A$ (A does not depend on n), that for every $f(t) \in L_p$

$$\lim_{n\to\infty} \int T^s_t f(t)\, x_n(t)\, dm(t) = f(s), \quad \lim_{n\to\infty} \int \tilde{T}^s_t f(t)\, x_n(s)\, dm(s) = f(t)$$

$$\lim_{n\to\infty} \int \tilde{T}^s_t f(t)\, x_n(t)\, dm(t) \in L_p$$

* The operation of generalized translation was originally introduced by J. Delsartes. The author applied the same notion in his papers ([6]).

The convergence is by the norm in L_p.

II. The linearity:

$$T_t^s[af(t)+bg(t)]=aT_t^s f(t)+bT_t^s g(t)$$

$f,\ g\in L_p$; a and b, complex numbers.

III. $T_s^r T_t^s f(t)=T_t^s T_t^r f(t)$ (associativeness).

IV. Commutativeness and normality:

$$T_t^s T_t^r f(t)=T_t^r T_t^s f(t),\quad \tilde{T}_t^s T_t^r f(t)=T_t^r \tilde{T}_t^s f(t)$$

V. The boundedness of operators T^s:

$$\left[\int |T_t^s f(t)|^p dm(t)\right]^{1/p}\leqslant C_p\left[\int |f(t)|^p dm(t)\right]^{1/p}$$

$$\left[\int |\tilde{T}_t^s f(t)|^p dm(t)\right]^{1/p}\leqslant C_p^*\left[\int |f(t)|^p dm(t)\right]^{1/p}$$

where C_p and C_p^* depend neither on s nor on $f(t)$.

VI. The continuity of the operators T^s: if $f(t)\in L_p$, it is possible for any $\varepsilon>0$, $s\in Z$ to find such a neighbourhood U of the point s that

$$\int |T_t^s f(t)-T_t^{s'} f(t)|^p dm(t)<\varepsilon^p,\quad \int |\tilde{T}_t^s f(t)-\tilde{T}_t^{s'} f(t)|^p dm(t)<\varepsilon^p$$

if $s'\in U$.

VII. The reality of the operators T^s: if $f(t)$ is a real function, then $T_t^s f(t)$ is also a real function.

Corollary I. $T_t^s\overline{f(t)}=\overline{T_t^s f(t)}$. In fact, in virtue of the reality and linearity of the operators T^s we have

$$T_t^s\overline{f(t)}=T_t^s\overline{[f_1(t)+if_2(t)]}=T_t^s f_1(t)-iT_t^s f_2(t)=\overline{T_t^s f(t)}.$$

Corollary II. Without reducing the generality, we may assume $x_n(t)$ from the condition I to be real. In fact, let $x_n(t)=x_n'(t)+ix_n''(t)$. From the reality of T^s and the condition I follows that for every real function $f(t)\in L_p$

$$\lim_{n\to\infty}\int T_t^s f(t)\,x_n''(t)\,dm(t)=0 \tag{1}$$

In virtue of the reality of T^s the equality (1) holds also for every complex function $f(t)$, which proves our assertion.

Corollary III. We shall now show that the following relation of symmetry holds: $T_s^r f(s)=T_r^s f(r)$ for every $f(s)\in L_p$. Put $T_t^r f(t)=K(t,\ r)$. In virtue of the first half of the condition IV we have $T_t^s K(t,\ r)=T_t^r K(t,\ s)$.

Hence, in virtue of the condition I,

$$\lim_{n\to\infty}\int T_t^s K(t,\ r)\,x_n(t)\,dm(t)=K(s,\ r)=\lim_{n\to\infty}\int T_t^r K(t,\ s)\,x_n(t)\,dm(t)=K(r,\ s)$$

Corollary IV. Let us show that the following condition takes place:

$$\tilde{T}_s^r T_t^s f(t)=T_t^s \tilde{T}_t^r f(t) \tag{2}$$

In virtue of the symmetry of T_t^s and the normality of the operators T^s,

$$\tilde{T}_s^r T_t^s f(t)=\tilde{T}_s^r T_s^t f(s)=T_s^t \tilde{T}_s^r f(s)=T_t^s \tilde{T}_t^r f(t)$$

Comparing the extreme terms of these equalities, we find (2). We shall also need the following identity:

$$\tilde{T}_u^r \tilde{T}_t^u T_t^s f(t)=\tilde{T}_t^u T_t^r T_t^s f(t) \tag{3}$$

It is evidently sufficient to show that for any function $h(t)\in L_p$ we have

$$\tilde{T}_u^r \tilde{T}_t^u h(t)=\tilde{T}_t^u T_t^r h(t) \tag{4}$$

and in virtue of (2) (replacing s by u),

$$\tilde{T}_u^r \int \tilde{T}_t^u h(t)\, g(t)\, dm(t) = \int h(t)\, \tilde{T}_u^r T_t^u g(t)\, dm(t) =$$

$$= \int h(t)\, T_t^u \tilde{T}_t^r g(t)\, dm(t) = \int \tilde{T}_t^u T_t^r h(t)\, g(t)\, dm(t)$$

Comparing the extreme terms, we find, since $g(t)$ is arbitrary, the required relation (4).

If Z is an Abelian group, then the simplest operation of generalized translation will be

$$T_t^s f(t) = f(t+s)$$

Here the group operation is written as addition. For the measure $m(\Delta)$ we must choose the invariant measure of Haar.

3. Denote by L the Banach space of all absolutely integrable functions $x(t)$ $(t \in Z)$ with the usual addition and multiplication by complex numbers and the norm

$$\| x \| = C_1 \int |x(t)|\, dm(t)$$

(C_1, from the condition V). Define in L the multiplication (composition) by

$$x \cdot y(t) = \int \tilde{T}_t^s x(t)\, y(s)\, dm(s)$$

Let us show that $x \cdot y = y \cdot x$. Let $x \cdot y = h_1(t)$, $y \cdot x = h_2(t)$ and $g(t)$ be an arbitrary function from L. In virtue of Fubini's theorem and the symmetry of T_t^s we have

$$\int h_1(t)\, g(t)\, dm(t) = \int\int \tilde{T}_t^s x(t)\, y(s)\, g(t)\, dm(s)\, dm(t) =$$

$$= \int y(s)\, dm(s) \int \tilde{T}_t^s x(t)\, g(t)\, dm(t) =$$

$$= \int y(s)\, dm(s) \int x(t)\, T_t^s g(t)\, dm(t) =$$

$$= \int y(s)\, dm(s) \int x(t)\, T_s^t g(s)\, dm(t) =$$

$$= \int x(t)\, dm(t) \int \tilde{T}_s^t y(s)\, g(s)\, dm(s) =$$

$$= \int g(s)\, dm(s) \int \tilde{T}_s^t y(s)\, x(t)\, dm(t) = \int h_2(t)\, g(t)\, dm(t).$$

Since $g(t)$ is arbitrary, $h_1(t) = h_2(t)$, q. e. d.

Again, by Fubini's theorem, we have $\| x \cdot y \| \leqslant \| x \| \cdot \| y \|$.

The sequence $x_n(t)$ from the condition I plays evidently the rôle of unity e in our ring.

Consider now the ring R consisting of elements $\zeta = \lambda e + x(t)$ (λ being a complex number, $x(t) \in L$) with the norm $\| \zeta \| = |\lambda| + \| x \|$. Arguing in the same way as in [1], we obtain the following results:

a) *The ring L forms in R a maximal ideal M_0, and under the homomorphism $R \to R/M_0$ the element $\zeta = \lambda e + x(t)$ is transformed into λ.*

All remaining maximal ideals of the ring R generate continuous characteristic functions of the operators T^s, i. e. functions $\varphi(t)$ satisfying the functional equation $T_t^s \varphi(t) = \varphi(s)\, \varphi(t)$.

b) *Every measurable characteristic function $\varphi(s)$ of the operators T^s satisfying for any function $x(t) \in L$ the relation*

$$\int \tilde{T}_t^s x(t)\, \varphi(t)\, dm(t) = \varphi(s) \int x(t)\, \varphi(t)\, dm(t) \tag{5}$$

generates a maximal ideal of the ring R different from M_0.

In fact, it suffices to put

$$(\zeta, \varphi) = \lambda + \int x(t)\, \overline{\varphi(t)}\, dm(t)$$

c) *Every measurable, bounded characteristic function of the operators T^s satisfying the relation* (5) *is continuous.*

Proof. Let

$$(\tilde{T}_t^s x(t), \varphi(t)) = \int \tilde{T}_t^s x(t)\,\overline{\varphi(t)}\,dm(t) = \overline{\varphi(s)} \int x(t)\,\overline{\varphi(t)}\,dm(t) = \overline{\varphi(s)}\,(x, \varphi)$$

Hence

$$\overline{\varphi(s)} = \frac{(\tilde{T}_t^s x(t), \varphi)}{(x, \varphi)}$$

which in virtue of the conditions VI proves the continuity of $\varphi(s)$.

d) *Every continuous characteristic function is generated by only one maximal ideal.*

The proof coincides with the proof of Theorem 4 in ([1]).

4. By $\mathfrak{M}$ we denote the bicompact topological space of all maximal ideals of the ring R. Inasmuch as the correspondence between the maximal ideals $\mathfrak{M} - M_0$ and the characteristic functions of the operators T^s is one-to-one, the aggregate of all characteristic functions of the operators T^s may be represented in the form of a function of two variables $\varphi(\alpha, t)$, $\alpha \in \mathfrak{M} - M_0$, $t \in Z$.

The functions $(\zeta, M) = \lambda + \int x(t)\,\overline{\varphi(\alpha, t)}\,dm(t)$ are continuous on $\mathfrak{M}$.

For the further development of the theory it is important to know when for any element $\zeta = \lambda e + x(t)$ there exists a conjugate element $\tilde{\zeta}$, *i. e.* such that $(\tilde{\zeta}, M) = \overline{(\zeta, M)}$. It turns out that

$$\tilde{\zeta} = \overline{\lambda} e + \lim_{n\to\infty} \int \tilde{T}_t^s\,\overline{x(t)}\,x_n(t)\,dm(t) = \overline{\lambda} e + \overline{x^*(t)}$$

In the first place we show that $\tilde{T}_t^r \varphi(t) = \overline{\varphi(r)}\,\varphi(t)$. We have

$$\int x(t)\,\tilde{T}_t^r\,\overline{\varphi(t)}\,dm(t) = \int T_t^r x(t)\,\overline{\varphi(t)}\,dm(t) =$$

$$= \varphi(r) \int x(t)\,\overline{\varphi(t)}\,dm(t) = \varphi(r)\,(x, \varphi)$$

Comparing the first and the last term, we find, since $x(t)$ is arbitrary, the required relation. Further, we have

$$(\tilde{\zeta}, M) = \tilde{\lambda} + \lim_{n\to\infty} \int\int \tilde{T}_t^s\,\overline{x(t)}\,x_n(t)\,\overline{\varphi(\alpha, s)}\,dm(t)\,dm(s) =$$

$$= \overline{\lambda} + \int x(t)\,dm(t) \lim_{n\to\infty} \int \overline{\varphi(\alpha, s)}\,T_s^t x_n(s)\,dm(s) =$$

$$= \overline{\lambda} + \int \overline{x(t)}\,\varphi(\alpha, t)\,dm(t) \lim_{n\to\infty} \int x_n(s)\,\overline{\varphi(\alpha, s)}\,dm(s) =$$

$$= \overline{\lambda} + \int \overline{x(t)}\,\varphi(\alpha, t)\,dm(t) = \overline{(\zeta, M)}$$

since the sequence $x_n(s)$ plays the rôle of unity in the ring R, and the residium by any maximal ideal for it is equal to 1.

Hence follows ([4]) that the functions (ζ, M) are everywhere dense in the space C of all continuous functions on $\mathfrak{M}$, the norm being $\|\psi\| = \sup|\psi(M)|$.

5. In the same way as in Raikoff's note ([5]) (the lemma) it may be shown that $\varphi(\alpha, t)$ is continuous in the aggregate of its variables α, t.

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Received
27.V.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

MATHEMATICS

ON A THEOREM IN THE THEORY OF PRIMES

By U. V. LINNIK

(Communicated by I. M. Vinogradoff, Member of the Academy, 30. XII. 1944)

§ 1. At the present state of the theory of primes, the distribution formulae with a power remainder are of considerable interest, *i. e.* the formulae in which the remainder does not exceed the principal term to a power $c_0 < 1$. A formula of such kind is given by I. M. Vinogradoff in his paper (1).

The number T of fraction of the form $\{bp^\alpha\}$; $p \leqslant N$ prime; $\alpha < 1$, under the condition

$$0 \leqslant \{bp^\alpha\} \leqslant \lambda \tag{1}$$

is expressed by the formula

$$T = \lambda\pi(N) + O(N^{1+\eta}\Delta), \quad \Delta = \left\{bN^{\alpha-1} + b^{-1}N^{-\alpha} + N^{-2/3\alpha}\right\}^{1/5} \tag{2}$$

I wish to show here that the formulae of this type can be also deduced from Mangoldt's classical formula (2) and from Hoheisel's theorem (2) on the zeros $\zeta(s)$; they are based on the facts concerning the «density» of zeros of $\zeta(s)$ in $\sigma > 1/2$, *i. e.* on the theorems of Bohr-Landau's type.

I shall prove here, in particular, the formula

$$Q(N) = \sum_{x=1}^{\sqrt{N}} \left\{\psi\{(x+1)^2\} - 2\psi\left\{\left(x+\frac{1}{2}\right)^2\right\} + \psi(x^2)\right\} \ll N^{63/64} \tag{3}$$

(ψ is Tchébichev's function) which corresponds to $b = 1$, $\lambda = 1/2$ in the formulae (1) and (2).

The general case is treated in the same manner.

§ 2. It is sufficient to prove the estimate (3) for the function $Q_1(N) = Q(N) - Q(N/4)$. Mangoldt's formula (2) gives:

$$\psi(y+h) - \psi(y) = h - \sum_{|\rho|<T} \frac{(y+h)^\rho - y^\rho}{\rho} + O\left(\frac{h}{y^2} + \frac{y}{T}\ln^2 y + \ln y\right) \tag{4}$$

Putting here $T = N^{17/32}$, we deduce from (4)

$$Q_1(N) \ll \left| \sum_{x=\sqrt{N/4}}^{\sqrt{N}} \sum_{|\rho|<N^{17/32}} \frac{(x+1)^{2\rho} - 2\left(x+\frac{1}{2}\right)^{2\rho} + x^{2\rho}}{\rho} \right| + N^{31/32}\ln^2 N \tag{5}$$

We make use now of the theorems of Van der Corput's type, namely, the Theorems 1 and 2 of the paper ([3]) by Segal, and obtain the lemmas:

Lemma 1. For $N^{15/32} \leqslant t_1 \leqslant N^{1/2}/10$, $\rho = \beta_1 + it_1$ we have

$$\left| \sum_{x=\sqrt{N/4}}^{\sqrt{N}} \frac{x^{2\rho}}{\rho} \right| < c_1 \frac{N^{\beta_1 + 1/32}}{|\rho|} \tag{6}$$

Lemma 2. Under analogous conditions, but with $N^{1/2}/10 \leqslant t_1 \leqslant N^{17/32}$ we have

$$\left| \sum_{x=\sqrt{N/4}}^{\sqrt{N}} \frac{x^{2\rho}}{\rho} \right| < c_2 \frac{H^{\beta_1}}{\sqrt{|\rho|}} \tag{7}$$

We divide the rectangle $\frac{1}{2} \leqslant \sigma \leqslant 1$, $N^{1/2}/10 \leqslant t \leqslant N^{17/32}$ into $\ll \ln^2 N$ narrow rectangles of the form $\beta \leqslant \sigma \leqslant \beta + 1/\ln N$, $N_0 \leqslant t \leqslant 2N_0$. By Hoheisel's theorem ([2]) the number of zeros in a rectangle of this type does not exceed $N_0^{1-(2\beta-1)^2} \ln^7 N$. Hence the sum of the expressions (7) extended over all ρ in our narrow rectangle does not exceed

$$c_3 N^{\beta + 17/32 (1/2 - (2\beta-1)^2)} \ln^7 N \tag{8}$$

The exponent assumes a maximal value for $\beta = \frac{25}{34}$ and equals then to $\frac{861}{1088}$. Hence the sum of (7) extended over the whole large rectangle does not exceed

$$c_4 N^{861/1088} \ln^{10} N \tag{9}$$

In the same way, dividing the rectangle $\frac{1}{2} \leqslant \sigma \leqslant 1$, $N^{15/32} \leqslant t \leqslant N^{1/2}/10$ into $\ll \ln^2 N$ rectangles of the type $\beta \leqslant \sigma \leqslant \beta + 1/\ln N$, $N_0 \leqslant t \leqslant 2N_0$ and using (6), we find that the sum over ρ of the expressions (6) does not exceed

$$N^{\frac{319}{480}} \ln^{10} N \tag{8'}$$

Taking into account the symmetry of the distribution of $\zeta(s)$ with respect to $\sigma = 1/2$ and $t = 0$, we deduce:

$$Q_1(N) \leqslant \left| \sum_{|\rho| \leqslant N^{15/32}} \sum_{x=\sqrt{N/4}}^{\sqrt{N}} \frac{(x+1)^{2\rho} - 2\left(x + \frac{1}{2}\right)^{2\rho} + x^{2\rho}}{\rho} \right| + N^{31/32} \ln^2 N \tag{10}$$

To work out the remaining sum over $|\rho| \leqslant N^{15/32}$, we remark that here we have $|2\rho/x| \ll N^{-1/32}$. Hence, by Newton's binomial formula,

$$\frac{\varphi(x, \rho)}{\rho} = \frac{(x+1)^{2\rho} - 2\left(x + \frac{1}{2}\right)^{2\rho} + x^{2\rho}}{\rho} = \frac{x^{2\rho}}{\rho} \left\{ \left(1 + \frac{1}{x}\right)^{2\rho} - 2\left(1 + \frac{1}{2x}\right)^{2\rho} + 1 \right\} =$$

$$= \frac{x^{2\rho}}{\rho} \sum_{r=2}^{32} \frac{P_r(\rho)}{x^r} + \vartheta \frac{B}{N}$$

where $|\vartheta| \leqslant 1$ is a constant, and $P_r(\rho)$ is a polynomial in ρ of degree r with fixed coefficients.

Now, using Theorem 1 of Segal's paper ([3]), we can prove

Lemma 3. For $\rho=\beta+it$, $r\geqslant 2$, $t<N^{15/32}$ we have

$$\left|\sum_{x=\sqrt{N/4}}^{\sqrt{N}} x^{2\rho-r}\right|<c_4\frac{N^{\beta-\frac{1}{2}(r-1)}}{|\rho|}$$

Summing the expression $\varphi(x,\rho)\rho$ with the aid of this formula and taking into account that the number of zeros ρ with $|\rho|\leqslant N^{15/32}$ is $O(N^{15/32}\ln N)$, we obtain that our sum does not exceed $N^{31/32}\ln N$. Taking into account (10), we obtain $Q_1 N\ll N^{31/32}\ln^2 N$, and hence, certainly,

$$Q(N)=\sum_{x=1}^{\sqrt{N}}\left\{\psi\{(x+1)^2\}-2\psi\left\{\left(x+\frac{1}{2}\right)^2\right\}+\psi(x^2)\right\}\ll N^{63/64}.$$

Received
30.XII.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

MATHEMATIQUES

GENERALISATION DE LA FORMULE FONDAMENTALE DE LA METHODE SYMBOLIQUE POUR LE CAS DES EQUATIONS DIFFERENTIELLES A COEFFICIENTS VARIABLES

Par I. SHTOKALO

(Présenté par S. L. Soboleff, de l'Académie, le 10. IV. 1944)

On sait que pour les équations différentielles à coefficients constants

$$dx/dt = Ax + F(t) \tag{1}$$

la possibilité d'appliquer la méthode symbolique de Heaviside a pour base le fait suivant: pour la fonction $F(t)$ de la forme

$$F(t) = Ce^{pt}$$

le système (1) admet une solution particulière de la forme

$$x = (pE - A)^{-1} Ce^{pt}$$

où E est la matrice unitaire.

Il est intéressant de généraliser ce résultat en considérant le cas d'équations différentielles linéaires à coefficients variables.

Considérons le système d'équations

$$dx/dt = A(t)\,x + Ce^{pt} \tag{2}$$

où $A(t)$ est une matrice n-dimensionnelle, x et C sont des vecteurs n-dimensionnels, et p est un certain nombre complexe.

Soit s_k les racines de l'équation algébrique

$$\mathrm{Det}\ \| pE - A_0 \| = 0$$

et soit

$$d(p) = \min | R(s_k - p) | \geqslant \alpha$$

Ces notations étant admises, nous avons:

Une matrice n-dimensionnelle étant donnée, on peut faire correspondre aux nombres positifs et d'ailleurs quelconques N et α un tel nombre $K > 0$ que le système d'équations différentielles

$$dx/dt = [A_0 + \varepsilon f(dt)]\,x + Ce^{pt}$$

où l'on a sur tout l'axe réel

$$| f(t) | \leqslant N$$

et les nombres complexes p et ε vérifient l'inégalité

$$| \varepsilon | \leqslant d(p)/K \tag{3}$$

possède une solution unique de la forme

$$x = \xi(t, p, \varepsilon)\, e^{pt}$$

Le facteur $\xi(t, p, \varepsilon)$ *est une fonction de* t, *bornée sur tout l'axe réel et analytique par rapport aux paramètres* p *et* ε *dans le domaine défini par l'inégalité* (3).

Dans le cas où $f(t)$ *est une fonction presque périodique,* $\xi(t, p, \varepsilon)$ *est aussi une fonction presque périodique de la variable* t *possédant la même base.*

On en déduit le théorème suivant qui permet d'étendre le résultat obtenu au cas des systèmes ne contenant pas un petit paramètre ε.

A chaque nombre positif N *on peut faire correspondre un* $L > 0$ *tel que la proposition suivante est vraie:*

Le système d'équations différentielles (2) *où l'on a sur tout l'axe réel les inégalités*

$$|A(t)| \leqslant N \text{ et } |R(p)| > L$$

possède une solution unique de la forme

$$x = \xi(t, p, \varepsilon)\, e^{pt}$$

où $\xi(t, p, \varepsilon)$ *est une fonction bornée par rapport à* t *dans l'intervalle* $(-\infty, +\infty)$ *et analytique par rapport aux paramètres* p *et* ε *dans le domaine* (3).

Dans le cas où $A(t)$ *est une fonction presque périodique,* $\xi(t, p, \varepsilon)$ *est aussi une fonction presque périodique de* t *ayant la même base.*

Il nous semble que les résultats obtenus peuvent être considérés comme point de départ pour étendre la formule fondamentale de la méthode symbolique au cas des équations différentielles linéaires à coefficients variables.

Manuscrit reçu
le 10.IV.1944.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

ASTRONOMIE

DETERMINATION DE LA TEMPERATURE DE COULEUR DES ETOILES SANS AIDE DES SOURCES DE COMPARAISON TERRESTRES

Par G. A. TIKHOV, Membre correspondant de l'Académie

1. Les défauts de la méthode usuelle. La comparaison des étoiles avec des sources terrestres de lumière a quelques défauts essentiels: 1) les sources terrestres ont une brilliance relative très faible dans l'ultraviolet; 2) il faut réaliser un contrôle continu et très soigneux pour maintenir la constance de leur température; 3) il faut affranchir les observations des étoiles de l'affaiblissement selectif de la lumière dans l'atmosphère terrestre qui change d'un jour à l'autre et d'une nuit à l'autre.

Tous ces défauts n'existent pas dans la méthode qui fait l'objet du présent article.

2. L'essentiel de la méthode proposée. On sait que l'indice de couleur des étoiles, c'est-à-dire la différence entre leurs magnitudes photographiques et visuelles, change de plus en plus lentement à mesure que la température augmente. Ceci est clair par le tableau 1, ou l'on trouve l'indice de couleur absolu C_a égal, par convention, à zéro pour $T = \infty$. Dans ce tableau T est exprimé en milliers de degrés de l'échelle Kelvin.

Tableau 1

T	2	5	8	11	14	20	40	70	100	∞
C_a	3.60	1.29	0.73	0.49	0.37	0.24	0.11	0.06	0.04	0

On voit d'ici qu'en prenant $T = \infty$ pour une étoile avec $T = 70000$, c'est-à-dire en prenant pour celle-ci $C_a = 0$, on ne fait dans C_a qu'une erreur de 0.06 du côté de la diminution. En prenant une telle étoile pour l'étoile de comparaison, on obtient pour toutes les autres étoiles C_a de 0.06 moindres que les vrais. Une telle erreur influence très peu les températures plus basses, et ce n'est que les températures élevées qui peuvent changer de milliers et même de dizaines de milliers de degrés.

Le tableau 2 fait voir l'erreur δT en T qui correspond à l'erreur de 0.05 magnitude stellaire dans l'indice de couleur.

Tableau 2

$T°$		2000	5000	8000	11000	14000	20000	40000
$\delta T°$	ou + 25°	+ 170	+ 500	+ 1100	+ 2000	+ 8000	+ 30000
	ou — 25°	— 160	— 400	— 750	— 1200	— 5000	— 9000

Ainsi l'idée primordiale de la méthode nouvelle conduisait à l'admission de $T = \infty$ pour l'étoile ayant le moindre indice de couleur dans la région mesurée.

3. La méthode statistique. Supposons que nous avons des indices de couleur d'un groupe assez considérable d'étoiles. Si l'on ajoute à ces indices une constante, jusqu'à présent inconnue C_∞, on obtiendra des indices de couleur absolus.

Pour trouver C_∞ agissons de la manière suivante. Partageons les étoiles en groupes avec les indices de couleur compris dans les limites, par exemple, de 0.1 de magnitude. Calculons ensuite pour chaque groupe le nombre d'étoiles et l'indice de couleur moyen. D'après ces données construisons un graphique, en mettant sur l'axe des abscisses l'indice de couleur moyen et sur l'axe des ordonnées—le nombre d'étoiles dans le groupe. On obtient ainsi la courbe de répartition des étoiles d'après l'indice de couleur.

Il est naturel qu'avec l'approche de l'indice de couleur vers les valeurs extrêmes d'un côté et de l'autre cette courbe approche de plus en plus de l'axe des abscisses.

Cependant la courbe ainsi construite est loin d'être géométriquement régulière, et l'on ne peut pas en faire de conclusions certaines. C'est pourquoi l'on calcule la courbe théorique en s'appuyant sur la courbe observée. L'abscisse du point de la courbe théorique, où elle s'approche, du côté des moindres indices de couleur, de l'axe des abscisses à la distance 1, donne la valeur la moindre possible de l'indice de couleur, puisqu'avec le moindre écart du côté de la diminution de l'indice de couleur le nombre d'étoiles est <1, c'est-à-dire il n'y en a plus et il ne peut pas en être. En d'autres termes, nous entrons ici dans le domaine de la température infiniment élevée. En retranchant l'abscisse de ce point limite de toutes les autres, nous obtenons les indices de couleur absolus de toutes les étoiles du groupe étudié.

4. Exemple de réduction d'un catalogue. Appliquons la méthode exposée à un groupe d'étoiles dans le Céphée, pour lequel on a imprimé, dans le volume 60 (2-me série) des Travaux de l'Observation de Poulkovo, un catalogue photométrique de 1958 étoiles, obtenu par M. D. Berg (Lavrova). On trouve dans ce catalogue les indices de couleur pour 749 étoiles et pour 273—les spectres. Les longueurs d'onde effectives pour les magnitudes photographiques et photovisuelles sont égales respectivement à 434 et à 580 mμ.

On trouve dans le tableau 3 les numéros des groupes, les indices moyens de couleur C et le nombre d'étoiles n dans chaque groupe. Les limites de l'indice de couleur dans le système du catalogue sont égales successivement:—0.40 et —0.31;—0.30 et —0.21, et ainsi de suite.

Tableau 3

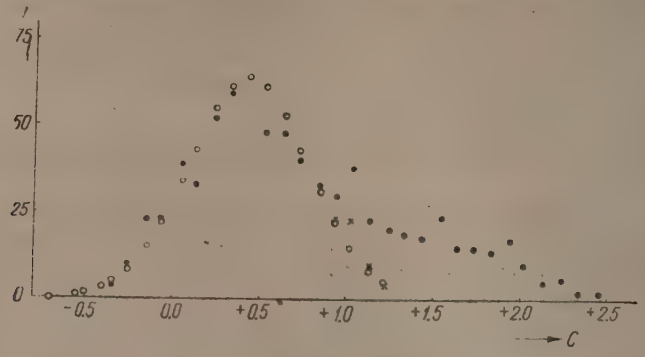
No. du groupe	C	n	No. du groupe	C	n
1	— 0.34	4	11	0.65	48
2	— .25	10	12	.74	40
3	— .14	23	13	.85	33
4	— .06	23			
5	+ .06	39	Points réfléchis		
6	.14	33			
7	.25	52	4′	0.94	23
8	.34	59	3′	1.02	23
9	.44	74	2′	1.13	10
10	.54	48	1′	1.22	4

Les données de ce tableau et les places avec $C>0.85$ sont représentées dans la figure, où les places observées sont indiquées par les points gras.

On ne s'intéresse dans ce cas que des étoiles de température très élevée. C'est pourquoi, pour le calcul de la courbe théorique de répartition, nous nous bornons du côté de C positifs de la valeur 0.85. On voit par le tableau 3 et par la figure que n atteint la valeur maximum à $C=0.44$. La droite $C=0.44$ peut être prise pour l'axe de symétrie de la courbe de répartition. Les déviations des indices de couleur de l'axe de symétrie peuvent être considérées comme des erreurs accidentelles d'observation. Nous aurons besoin de l'erreur quadratique moyenne d'une observation, ce qui demande d'avoir une courbe complète de répartition. Cependant les points aves $C>0.9$ deviennent déjà manifestement de la symétrie avec les points qui se trouvent à gauche de l'axe de symétrie; c'est pourquoi nous ne pouvons pas nous en servir, et nous construisons les images de quatre premiers points réfléchies par l'axe de symétrie. Ces images sont représentées dans la figure par de petites croix.

Ainsi, pour la construction de la courbe théorique de répartition nous

avons dans le tableau 3 les 13 premiers groupes directement observés et les 4 derniers groupes réfléchis.



On trouve dans le tableau 4 les étapes principales du calcul d'après ces 17 groupes. Les places calculées sont représentées dans la figure par de petits cercles.

Tableau 4

No. du groupe	C	$\Delta = 0.44 - C$	n	$n\Delta^2$	$t \frac{\Delta}{\varepsilon}$	y_t	$\nu = ky_t$
1	—0.34	+0.78	4	2.43	2.28	0.029	5
2	— .25	.69	10	4.76	2.01	.052	8
3	— .14	.58	23	7.74	1.69	.095	15
4	— .06	.50	23	5.75	1.46	.138	22
5	+ .06	.38	39	5.63	1.11	.214	34
6	.14	.30	33	2.97	0.88	.270	43
7	.25	.19	52	1.88	.55	.342	55
8	.34	.10	59	0.59	.29	.382	61
9	.44	.00	74	.00	.00	.399	64
10	.54	— .10	48	.48	.29	.382	61
11	.65	— .21	48	2.12	.61	.330	53
12	.74	— .30	40	3.60	.88	.270	43
13	.85	— .41	33	5.55	1.20	.194	31
			Groupes réfléchis				
4′	0.94	—.50	23	5.75	1.46	.138	22
3′	1.02	—.58	23	7.74	1.69	.195	15
2′	1.13	—.69	10	4.76	2.01	.152	8
1′	1.22	—.78	4	2.43	2.28	0.029	5
Somme			546	64.18		3.411	545

Pour la valeur quadratique moyenne d'un Δ on trouve $\varepsilon = \pm 0.343$. On calcule ensuite $t = \left|\frac{\Delta}{\varepsilon}\right|$ et par les tableaux usuels de la théorie des erreurs on trouve la quantité $y_t = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$ proportionnelle à la probabilité de l'apparition d'une telle ou telle valeur de Δ. Après cela on calcule le facteur $k = \Sigma n / \Sigma y_t = 160.1$, et l'on trouve le nombre théorique d'étoiles $\nu = ky_t$.

Pour $\nu = 1$ on a $y_t = 0.0062$. On trouve ensuite $t = 2.88$ et, par les tableaux d'erreurs, $\Delta = 0.99$. Enfin on a $C_\infty = -0.55$.

C'est cette valeur qu'on peut adopter, dans le système du catalogue étudié, pour l'indice de couleur de $T = \infty$. Par conséquent, pour obtenir les indices de couleur absolus, on doit retrancher -0.55 de tous les C de façon que $C_a = 0$ pour $T = \infty$.

On donne dans le tableau No. 5 les valeurs ainsi trouvées de C_a et T peur les classes spectrales principales du catalogue étudié.

Tableau 5

Spectre	n	Limites C_a	Limites T	T moyenne
			en milliers de degrés	
B0	15	0.34—1.09	16.7—6.5	8.9
A0	46	.40—1.36	14.5—5.4	8.3
F0	11	.71—1.32	9.1—5.5	7.35
G0	10	.72—1.46	9.0—5.1	6.45
K0g	10	1.38—2.13	5.3—3.65	3.7
Mag	3	2.35—3.13	3.35—2.6	3.0

La valeur de T obtenue ici pour les étoiles G0 se trouve dans un bon accord avec la valeur d'un usage général.

La méthode proposée possède une qualité tout-à-fait exclusive. Les températures des étoiles obtenues par cette méthode ne dépendent nullement des propriétés d'absorption de notre atmosphère à la seule condition que le groupe étudié occupe une aire suffisamment petite. En effet, si, par exemple, la transparence dans les rayons photographiques change de δm, alors toutes les étoiles du groupe étudié donneront l'indice de couleur changé de cette même fraction de magnitude stellaire (m). Par conséquent, toute la courbe de la figure sera déplacée de la même quantité à droite ou à gauche, et C_∞ changera d'une façon identique avec tous les C. Il en suit que les valeurs C_a resteront les mêmes.

Nous nous affranchissons donc complètement de l'influence de l'atmosphère, et les observations se font comme en dehors de celle-ci.

La méthode proposée peut être perfectionnée et étendue sur le nombre voulu de régions du spectre.

J'espère de pouvoir consacrer une partie de mon temps à l'étude de cette question.

Manuscrit reçu
le 10.X.1944.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

HYDRODYNAMICS

A FUNDAMENTAL LAW OF GAS MOTION THROUGH A POROUS MEDIUM

By L. S. LEIBENSON, Member of the Academy

The author appears to have been the first to discuss the problem of the laminar and turbulent motion of gas through a porous medium. He gave its solution in his earlier papers, published in 1929 and 1930 (1). The power law he had derived for the filtration of gas through a porous medium was set forth in his monograph (2) on underground hydraulics in 1934. In the present paper this law of filtration is given in a general form. Use is made throughout of the notation and results reported in chapter XIII of the monograph (2).

The formulae (14), (15), (25) and (26) give the following expression of the power law (1) for the rate of filtration of a gas through a porous medium along the x-axis, on the assumption that the motion is turbulent and the process of the alteration of the gas state is polytropic:

$$\gamma u = A_1 f(m)\, \varphi d^{3s-1} \left(\frac{\partial P}{\partial x}\right)^s \tag{1}$$

$$\varphi = \mu g\, (\beta g \mu^2)^{-s} \tag{2}$$

If the structure of the medium corresponds to Slichter's scheme, we have

$$f(m) = f_1 = (1-m)^{-s}\, (n)^{(3s+1)/2} \tag{3}$$

and if it corresponds to the scheme of Coseni, we have

$$f(m) = f_2 = m^{3s}\, (1-m)^{1-2s} \tag{4}$$

A_1 is a number that depends on the structure of the medium and the exponent of the polytrope

$$p = (\beta\gamma)^{k_1} \tag{5}$$

It can be obtained by computation, but it is better to determine it by experiment; γ is the specific weight of the gas; ρ is the density; g is the acceleration of gravity, the last three quantities beirg connected by the relation

$$\gamma = g\rho \tag{6}$$

μ is the coefficient of the absolute viscosity of the gas; m is the porosity of the medium; n is the clear space; d is the so-called mean effective diameter of a particle of the medium; β is the gas constant. The exponent s is confined within

$$1/2 \leqslant s \leqslant 1$$

$s = 1$ corresponds to a laminar filtration: $s = 1/2$ is the customarily adopted value, but Smreker propose for the turbulent filtration of water to put $s = 2/3$.

We denote further

$$P = p^{1+\frac{1}{k_1}} \tag{7}$$

and adopt the notations

$$\mathrm{Re} = \frac{\rho\, du}{\mu} \tag{8}$$

$$\sigma = f(m) \tag{9}$$

$$\omega_1 = \frac{d^3}{\beta g \mu^2} \frac{\partial P}{\partial x} \tag{10}$$

where Re is Reynolds' number; σ is a dimensionless quantity described by the name of Slichter's number; and ω_1 is a dimensionless quantity, too.

The introduction in (1) of expressions (2) and (6) will give our power law, in the adopted notation, the form

$$\mathrm{Re} = A_1 \sigma \omega_1 s \tag{11}$$

Introducing (7) into (10) we obtain

$$\omega_1 = \left(1 + \frac{1}{k_1}\right) \omega \tag{12}$$

where ω is a dimensionless quantity

$$\omega = \frac{\rho d^3}{\mu^2} \frac{\partial p}{\partial x} \tag{13}$$

which will be called filtration number.

Let us substitute for the number A_1 a number A by means of the relation

$$A = A_1 \left(1 + \frac{1}{k_1}\right)^s \tag{14}$$

Then (11) finally takes the form

$$\mathrm{Re} = A \sigma \omega_s \tag{15}$$

This form of the power law generalizes spontaneously into

$$\mathrm{Re} = F(\omega) \tag{16}$$

which is the most general form of the filtration law for motion in one dimension (along x-axis).

Conversely, we have from (16) the relation

$$\omega = \Phi(\mathrm{Re}) \tag{17}$$

which, to put it into words, means that the number of filtration is a function of Reynolds' number.

The relations (15), (16) and (17) are valid for an incompressible fluid if $\rho = \mathrm{const}$.

Formula (16) can be obtained from the principle of dimension.

Assume, in fact, that

$$\mathrm{Re} = \sum_{\alpha} B_\alpha \left(\frac{\partial p}{\partial x}\right)^\alpha d^\beta \rho^\gamma \mu^\delta \tag{18}$$

where every B_α is a number.

Obviously, each term of the right-hand side of (18) is a dimenionless quantity, and so we have

$$\alpha + \gamma + \delta = 0, \quad -2\alpha + \beta - 3\gamma - \delta = 0, \quad -2\alpha - \delta = 0$$

From this

$$\beta = 3\alpha,\ \gamma = \alpha,\ \delta = -2\alpha \tag{19}$$

After (19) is introduced in (18) we shall by virtue of (13) obtain

$$\mathrm{Re} = \sum B_\alpha \omega^\alpha \tag{20}$$

which is in accord with (16).

Let us adopt the notations

$$D = \frac{d^3}{\mu^2} \tag{21}$$

$$q(p) = \int \rho(p)\, dp + \mathrm{const} \tag{22}$$

$$\Delta_1 q = \left(\frac{\partial q}{\partial x}\right)^2 + \left(\frac{\partial q}{\partial y}\right)^2 + \left(\frac{\partial q}{\partial z}\right)^2 \tag{23}$$

$$\theta(p) = \frac{F(D\sqrt{\Delta_1 q})}{\sqrt{\Delta_1 q}} \tag{24}$$

supposing that ρ is a known function of pressure p. ρ can, for instance, be determined from (5) in the case of a polytropic process. For the components of the velocity of filtration taken along the coordinate axes we have

$$\frac{\rho du}{\mu} = \theta \frac{\partial q}{\partial x},\ \frac{\rho dv}{\mu} = \theta \frac{\partial q}{\partial y},\ \frac{\rho dw}{\mu} = \theta \frac{\partial q}{\partial z} \tag{25}$$

In the case of incompressible fluid

$$q = \rho_0 p + \mathrm{const} \tag{26}$$

where ρ_0 is the constant density of the fluid. The formulae (25) are to be introduced in the continuity equation in the porous medium

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + m\frac{\partial \rho}{\partial t} = 0 \tag{27}$$

which is a differential equation in partial derivatives of the second order with respect to q. For from (22) we have

$$\rho = f(q) \tag{28}$$

where f_q is a known function of q.

Integration of this equation under certain boundary conditions and a given initial condition solves the problem of the movement of a gas or an incompressible fluid through a porous medium, whether this movement is a laminar or a turbulent one.

In the particular case of the power law (15) we shall have

$$\frac{F(D\sqrt{\Delta_1 q})}{\sqrt{\Delta_2 q}} = \frac{AcD^s}{(\Delta_1 q)^{1-s/2}} \tag{29}$$

which leads to the formulae obtained earlier[3].

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Received
28.I.1945.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

PHYSICS

ON FORCES ACTING UPON DIAMAGNETIC BODIES

By V. K. ARKADIEV, Corresponding Member of the Academy

P. Lebedev in 1911 pointed out the principle of absolute astaticism of a mobile magnetic system. His method was to add diamagnetic parts to a suspended paramagnetic body (1). Recently Braunbek (2) has studied a similar question in the general form. Proceeding from the usual formulae (3) he computed the field strength necessary for the magnetic forces to overcome the gravitational pull of the Earth on a tiny paramagnetic body vd, weighing several milligrams, so as to make it float in the air.

In fact, the force of translation is

$$f = v\varkappa_0 H \frac{dH}{dr}$$

For ordinary paramagnetic bodies we find

$$H\frac{dH}{dr} = \frac{gd}{\varkappa} = \frac{g}{\gamma}$$

where $\varkappa$ is specific susceptibility. The quantity $\varkappa$ being small, the field H and its gradient dH/dr have to be taken very large. In Braunbek's experiments the necessary field strength was about 23 000 oersteds. In a vacuum a suspension perfectly free from friction can be achieved by attaching the requisite object to a diamagnetic sphere floating between the ends of a big electromagnet.

With strongly diamagnetic bodies—such are superconductors, for instance—a suspension upheld by a strong lifting power can be created in a small magnetic field.

Reversing the problem we can make a magnet float over a superconductor.

The superconductor is looked upon as a body with a magnetic permeability $\mu = 0$. No force line of the magnetic field enters the superconducting plate. This means that near the plate the lines of the magnetic flux from the magnet are so deformed as to have the plane surface of the plate for boundary. This is only possible if that plate has a magnetic image of the magnet formed in it which faithfully reproduces the magnet in all proportions and is symmetrical to it as regards their position with respect to the plate's surface. Under these conditions the magnetic flux is compressed unilaterally. Owing to this, a superconducting plate should repel any magnet. When the repelling force is equal to the weight of the magnet, the latter floats over the plate. If a straight magnet of length l is perpendicular to the surface of the plate and the distance h between it and the plate is great in comparison with l, one can compute the repelling force as follows.

The field at the axes of the magnet at a distance of $2h = r$ from it is known to be

$$2M/r^3 \qquad (1)$$

where $M = ql$ is the magnetic moment of the magnet, q being its pole strength.

The action of this field upon the magnet is

$$f_{\perp} = 2Mq\left(\frac{1}{r^2} - \frac{1}{(r+l)^3}\right) = \frac{6M^2}{r^4} \qquad (2)$$

If the magnet is parallel to the plate, then taking r to be large in comparison with l we shall have

$$f_{\parallel} = \frac{3M^2}{r^4} \qquad (3)$$

Obviously $f_{\parallel} < f_{\perp}$. Therefore a magnet falling on a superconducting surface will assume a position parallel to it. The force $f_{\parallel}$ in the case of a floating magnet is equal to its weight vgd. Putting $M = vI$, we find that

$$h = \frac{1}{2}\sqrt[4]{\frac{3vI^2}{gd}} \qquad (4)$$

if the magnet is floating at a great height, so that l can be neglected in comparison with $h = \frac{r}{2}$.

As the volume v is proportional to the third power of linear dimensions: $v = ka^3$, we have

$$\frac{h}{a} = \frac{r}{2}\sqrt[4]{\frac{3kI^2}{agd}} \qquad (5)$$

This means that the relative height of floating is the greater, the smaller the dimensions of the magnet. Setting $I = 300$, $d = 7$ and $g = 981$, we find

$$\frac{h}{a} = 1.11\sqrt[4]{\frac{k}{a}}$$

Of course, this formula is only true for large h/a. Turning to molecular dimensions and putting $k = 1$ and $a = 10^{-8}$ cm, we obtain

$$h/a = 111$$

This shows that the silver atoms in Stern-Gerlach's experiment will not alight on a superconducting surface when falling slowly towards it. They will float in the air at a height of about one hundredth of a micron. The field strength at the surface of the superconductor M/h^3 is some hundredths of an oersted.

The most simple case is when the magnet field H in the neighbourhood of the plate does not exceed the critical field of the superconductor H_{cr}. In order to estimate the field H experiments were carried out with two Ni — Fe — — Al magnets. One of the magnets was made to float over the other, being prevented from sideslipping by guides. The other magnet played the rôle of the image under the horizontal surface of a superconductor. In this way it was found that a magnet weighing 1 to 8 g and measuring 1.5 to 3 cm in length would float over a like magnet at a height of about 1 cm (as measured between the axes of the magnet), irrespective of the dimensions. The pole strength is

$$q = r\sqrt{pg/2}$$

where p is the mass of the magnet.

The maximum value of the field H in the middle when the magnets are drawn together, that is when a magnet of length l lies on the surface of a superconductor, is

$$H = \frac{16}{l^2}r\sqrt{\frac{pg}{2}}$$

Thus, we find $H = 111$—157 oersteds.

Upon floating the field strength is lower. It is seen to be then by far smaller than the critical field of lead, which attains a value of 700 oersteds for 2.7 K.

The phenomena described here could not possibly have taken place if the magnetic field of the image had produced an appreciable demagnetizing influence on the magnet itself. So the magnet must possess a sufficient coercive force H_c.

An iron-nickel-aluminium steel with $H_c = 500$ oersteds will serve the purpose quite well. On the other hand, a tungsten magnet or one made of carbon steel is demagnetized on approaching a superconducting surface to such an extent that it cannot overcome the gravitational attraction and is unable to float unless its dimensions are very small. For a carbon steel magnet to float in the air it must have a size of 0.5×9 mm and a weight of several milligrams.

By means of a long straight magnet set upright an umbrella-like shell made of superconducting material can be kept floating in the air. A cap rotating without friction on the pivot of a compass may serve here as an illustration. Maxwell [4] gave a diagram which can be used in calculations if the pole of the magnet is supposed to be a point charge. The diagram represents the lines of force near two charges of like signs, one of which is four times as strong as the other. The stronger charge causes the lines of force from the weaker one to bend backwards so that they are bounded by a surface of the sixth order, *viz.* the surface of a blunt cone having an apex angle of about 107° and a radius of curvature of about $3h$. This surface is just an instance of our umbrella-like shell. The strong charge is the magnetic image of the weak charge. It is twice as far removed from the shell as the point-pole of the magnet. The repulsion between the pole and the shell is $4q^2/9h^2$; between the pole and the plane, $q^2/4h^2$. The first repelling force is 16/9 times as great as the second. When such a surface is rotating on its axis, which is also the axis of the magnet, the eddy currents forming in it need not change their position in the superconductor as they do in the case of a magnet hovering over a plane or a slightly concave surface.

In designing a cylindrical cap $4h$ in diameter one can add the field of the point pole q to the homogeneous field $H = q/h^2$, where h is distance from the bottom. Supposing the radius of curvature of the bottom to be 1.6 h cm, we have $f = q^2/h^2$. At the same distance from the plane the force is seen to be four times smaller (the diagram has been built by V. M. Goitannikov).

The floating of tiny magnets over a superconducting surface can be used in the study of superconducting inclusions in a metal at low (perhaps not even very low!) temperatures. The microscopic magnets will slip to the portions of ordinary conductivity. This method of observation is similar to the creation of the so-called Bitter's patterns from emulsions made of magnetic powders. In this way magnetic domains can be distinguished from superconductivity domains.

The considerations reported here were for the first time made publicly known by the author on December 12, 1944, in an address to a conference of the Moscow State University which was devoted to the discussion of recent problems in science. My desire to have them put to the test came soon to realization owing to the kindness of P. L. Kapitza, Director of the Institute of Physical Problems, Moscow, to whom I would fain offer my heartiest thanks. At his Institute experiments were arranged in January, 1945. A magnetized niferalium bar 4×4 mm in cross-section and 1 cm long was thrown upon a concave lead disc about 4 cm in diameter placed at the bottom of a Dewar vessel filled with liquid helium. After several elastic bounds along the vertical and many rapid complicated oscillatory movements the magnet assumed a horizontal position at a height of many millimeters above the lead. But eventually, owing to the increase in temperature caused by the evaporation of helium, the disc lost its superconductivity and the magnet settled quietly upon it.

Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

PHYSICS

SOUND RECEIVER IN MOTION

By D. BLOKHINTZEV

(Communicated by S. I. Wawilow, Member of the Academy, 17. V. 1944)

To put a simple example of the general theory of propagation of sound in a moving medium as developed by the author in an earlier paper (1), we may now discuss the reception of sound by a receiver considered to be a potential flow, which implies that the receiver is of streamline form and that the receiving diaphragm is placed somewhere in the front, outside the influence of the vortices that spring off from the body of the receiver. To begin with, we are to solve the following problem. Suppose an arbitrary number of fixed solid bodies to be located in a sound field described by the potential $\psi_0 e^{i\omega t}$ (ω, frequency of vibrations; t, time). This potential satisfies

$$\Delta\psi_0 + k^2\psi_0 = 0, \qquad k = \omega/c \tag{1}$$

(c, velocity of sound). At the surface of the solid bodies we have also the boundary condition $\partial\psi_0/\partial n = 0$. Further a flow with potential Φ_0 is supposed to be moving past these bodies:

$$\Delta\Phi_0 = 0, \quad \partial\Phi_0/\partial n = 0 \qquad \text{(at the surface of the bodies)} \tag{2}$$

Given ψ_0 and Φ_0, let us find the sound field potential φ in the presence of the flow. If the velocity of the flow $v \ll c$, then φ will satisfy the equation (1)

$$\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = \Delta\varphi + \frac{2}{c^2}\vec{\nabla}\Phi_0 \cdot \vec{\nabla}\frac{\partial\varphi}{\partial t} + O(v^2/c^2) \tag{3}$$

Putting $\varphi = \varphi_0 e^{i\omega t}$ we get

$$\Delta\varphi_0 + k^2\varphi_0 + 2ik\vec{\nabla}\Phi_0 \cdot \vec{\nabla}\varphi_0 = 0 \tag{4}$$

This equation is solved with an accuracy up to v/c by the substitution

$$\varphi_0 = \psi_0 e^{-i\frac{k}{c}\Phi_0} \tag{5}$$

Also the boundary condition $\partial\varphi_0/\partial n = 0$ (at the surface of the solid bodies) is fulfilled, since $\partial\Phi_0/\partial n$ and $\partial\psi_0/\partial n$ are zero at these surfaces. So the relation (5) solves the problem. Let us now find the pressure at the surface of the receiver. In virtue of Bernulli's theorem

$$p = \text{const} - \frac{\rho v^2}{2} + \rho\frac{\partial\varphi}{\partial t} - \rho\vec{\nabla}\Phi_0 \cdot \vec{\nabla}\varphi \tag{6}$$

The sound pressure π is obtained from (5) and (6) in the form

$$\frac{\pi}{\rho} = (i\omega\psi_0 - \vec{\nabla}\Phi_0 \cdot \vec{\nabla}\psi_0)\, e^{-i\frac{k}{c}\Phi_0} + O(v^2/c^2) \tag{7}$$

Far away from the body this formula only expresses the fact that π does not depend on the motion of the body (to become convinced of that one has to take the Doppler effect into account). But near the body of the receiver, things may be different. In fact, at a distance great enough from the body

the quantity $\dot{\psi}_0$ is equal to $\pi_0/i\omega\rho$, where π_0 is the pressure of the passing wave at infinity. Near the body this quantity is of the same order of magnitude. $\vec{\nabla}\dot{\psi}_0$ near the body has the same order of magnitude with $\frac{\pi_0}{i\omega\rho}\frac{1}{d}$, where d denotes the dimensions of the body. By virtue of this the second term in (7) corresponds approximately to $\frac{v}{d}\frac{\pi_0}{\rho\omega}$ (where v is the tangential velocity of the flow). This second term is larger than the first if

$$\frac{v}{d}\frac{1}{\omega} \gg 1, \qquad \text{that is if} \qquad \frac{v}{c} \gg 2\pi\frac{d}{\lambda} \tag{8}$$

Obviously, the signal received will be amplified if only its wave-length is large enough, and the dimensions of the body of the receiver d are sufficiently small.

Considerable interest attaches to the question as to how the receiver will behave in the case of supersound velocity of motion. In a very primitive way this problem has been discussed at the German Akademie der Luftfahrtforschungen ([2]). Clearly, no sound signal travelling behind the receiver will ever reach it in this case. And one that is moving before is bound to cross the front of the shock wave which separates the medium undisturbed by the motion of the body from that disturbed. As the shock wave front travels at a rate higher than the velocity of sound in a calm medium, no reflected wave can possibly form in spite of the fact that the properties of the medium change jumpwise at the surface formed by the front of the shock wave. It may seem on first consideration that we have come across a paradox. How indeed should the usual boundary conditions be satisfied in the absence of reflected wave? But if the problem cannot be solved completely at this juncture, still some answer can be provided.

At the surface of the shock wave which constitutes a jump in density, irreversible processes take place. Therefore, the change of entropy should be taken into account. Suppose we have a simple straight jump of density and a sound wave falling towards it. Let the jump be moving with velocity V in the direction of negative x. Denote by U the rate of movement of the gas; and by ξ, the velocity of sound vibrations. In a system of coordinates with respect to which the jump is at rest, the velocity of the gas movement is $u = U - V$, and the equations for the propagation of sound run thus ([3]):

$$\frac{\partial\xi}{\partial t}+u\frac{\partial\xi}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x},\quad \frac{\partial\delta}{\partial t}+u\frac{\partial\delta}{\partial x}+\rho\frac{\partial\xi}{\partial x}=0,\qquad \frac{\partial\sigma}{\partial t}+u\frac{\partial\sigma}{\partial x}=0 \tag{9}$$

where p is pressure, ρ is density of the medium, δ is change in density due to the sound wave, σ is change of entropy S, $\frac{\partial p}{\partial x}=\left(\frac{\partial p}{\partial\rho}\right)_S\frac{\partial\delta}{\partial x}+\left(\frac{\partial p}{\partial S}\right)_\rho\frac{\partial\sigma}{\partial x}$. Further $\left(\frac{\partial p}{\partial\rho}\right)_S=c^2$ is the square of the adiabatic velocity of sound: $(\partial p/\partial S)_\rho = \frac{\gamma-1}{r}p$ (for an ideal gas); γ is the specific heat ratio c_p/c_v, and r is the gas constant. From the relations

$$(\omega+uk)\,\xi+\frac{c^2}{\rho}\delta=-\frac{\gamma-1}{r}\frac{p}{\rho}k\sigma,\quad (\omega+uk)\,\delta+k\rho\xi=0,\qquad (\omega+uk)\,\sigma=0 \tag{10}$$

assuming that the quantities ξ, δ, σ are proportional to $e^{i(\omega t-lx)}$, we get

$$k=\frac{\omega}{\pm c-u},\quad \xi\mp\frac{c}{\rho}\delta,\quad \sigma=0 \tag{11}$$

or

$$k=-\frac{\omega}{u},\quad \xi=0,\quad \sigma=\frac{r\gamma}{\rho(\gamma-1)}\delta \tag{11'}$$

The first solution is an ordinary adiabatic sound wave, the second, an entropy wave which is simply a transfer of a periodically varying entropy by

a medium moving with velocity u. It is exactly such periodical variations of entropy that are caused by a sound wave reaching the jump of density. We have to make the solutions join at the density jump. For this purpose we must use Hugoniot – Rankin conditions

$$u\rho = u_0\rho_0, \quad \rho u^2 + p = \rho_0 u_0^2 + p_0, \quad w + \frac{u^2}{2} = w_0 + \frac{u_0^2}{2} \tag{12}$$

where w is a heat function; for the ideal gas $\gamma/(\gamma-1)\cdot p/\rho$. These conditions are known to express the law of conservation of matter, momentum and energy. On the passage of sound wave these conditions are subject to variation, because all quantities receive small increments. The rate of motion of the jump also suffers a change, so that

$$\delta u = \delta U - \delta V = \xi - \eta \quad (\eta = \delta V)$$

$$\delta\rho = \delta, \quad \delta S = \sigma, \quad \delta p = \left(\frac{\partial p}{\partial \rho}\right)_S \delta + \left(\frac{\partial p}{\partial S}\right)_\rho \sigma, \quad \delta w = \left(\frac{\partial w}{\partial \rho}\right)_S \delta + \left(\frac{\partial w}{\partial S}\right)_\rho \sigma$$

In the undisturbed medium before the jump (the quantities relating to it are indicated by a zero) there is only a falling sound wave for which $\sigma_0 = 0$, $\frac{c_0}{\rho_0}\delta_0$. Behind the jump there is a complete change in density $\delta = \delta' + \delta''$, where δ' relates to the traversing sound wave ($\xi' = -\frac{r}{\rho}\delta'$, $\sigma = 0$), and δ'' relates to the entropy wave for which $\xi'' = 0$, $\sigma = -\frac{r\gamma}{\rho(\gamma-1)}\delta''$. Varying the conditions (12) and expressing all variations by δ_0, δ', δ'', we should get the required boundary conditions

$$(u-c)^2\delta' + u^2\delta'' = (u_0 - c_0)^2\delta_0 \tag{13}$$

$$\left[u^2\frac{\rho}{\rho_0} + c^2 - cu\left(\frac{\rho}{\rho_0}+1\right)\right]\delta' + \left[u^2\frac{\rho}{\rho_0} - \frac{c^2}{\gamma-1}\right]\delta'' =$$

$$= \left[u_0^2 + c_0^2\frac{\rho}{\rho_0} - c_0 u\left(\frac{\rho}{\rho_0}+1\right)\right]\delta_0; \quad u,\ u_0 < 0,\ u_0 = -V \tag{14}$$

By means of (12) we can express δ'/δ_0, δ''/δ_0 through the jump of pressure $p - p_0$. In its general form the expression is somewhat cumbersome. It will suffice here to give the result for small jumps $\varepsilon = (p-p_0)/p_0 \ll 1$, and for large jumps $\varepsilon = (p-p_0)/p_0 \gg 1$.

In the first case we have

$$\frac{\delta'}{\delta_0} = 1 + \frac{13-3\gamma}{8\gamma}\varepsilon + \ldots, \quad \frac{\delta''}{\delta_0} = \frac{\gamma-1}{2\gamma}\varepsilon + \ldots \tag{15}$$

From this the pressure of sound wave (π') is obtained as

$$\frac{\pi'}{\pi_0} = 1 + \frac{5(\gamma+1)}{8\gamma}\varepsilon + \ldots \tag{16}$$

In the second case ($\varepsilon \gg 1$)

$$\frac{\pi'}{\pi_0} = \frac{1}{\gamma}\frac{1}{1+2\left(\frac{\gamma-1}{2\gamma}\right)^{1/2}}\varepsilon + \ldots \tag{17}$$

As for the entropy wave, its pressure π'' is, according to (10), always zero, because $\omega + uk = 0$.

Thus, there will be no reflection of sound wave from the shock wave surrounding a moving receiver. But two passing waves will arise which will reach the receiver. These two waves have equal frequencies but different speeds. One of the waves does not bring about any change of pressure.

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Received
17.V.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

TECHNICAL PHYSICS

ON COLOUR SELECTION IN BLACKOUT LIGHTING

By A. WOLKENSTEIN

(Communicated by S. I. Wawilow, Member of the Academy, 21. VI. 1944)

1. The expediency of employing coloured lighting in blackout has been the subject of prolonged discussion ever since the first World War ([1–7]). Heretofore, however, this discussion, which has been especially lively during the present war, has not led to any universally accepted and sufficiently convincing conclusion. Hence the colour problem in blackout lighting is solved differently in different countries. The reason for such situation is the complexity and inherent contradictions of the problem as a whole and the fact that it was not always tackled correctly.

In our research we have started from the following points

a) Blackout lighting must be such that persons within the blacked out premises should be able to work or at least find their way about. At the same time the illuminated territory should not be visible to observers in enemy planes at a tactical operative distance from the blacked out premises.

b) The illuminated territory is, on the one hand, a field of adaptation for the eyes of persons within it and, on the other, an object of observation for enemy flyers.

c) The visual tasks of persons on the ground differ from those of the aerial observer. Persons within the bounds of the illuminated territory must be able either to perform some work (in this case «working illumination» is provided) or to find their way about (in this case «orientation illumination» is provided). The aerial observer, on the other hand, has only one visual task in either case, namely to detect the blacked out objective by its illumination.

d) For each of these two types of blackout lighting the visual functions characterizing the visual work of the flyer and the person on the ground should be considered in conjunction with one another. Here it should be kept in mind that the eye of the person on the ground is adapted to darkness in «orientation illumination» and adapted to light in «working illumination».

2. The problem of blackout lighting reduces to the creation of such conditions on the ground under which the illumination at the pupil of the eye of the enemy flyer will be below its threshold value for any spectral composition of the light.

Comparing the spectral sensitivity of the eye adopted by the International Illumination Committee as the mean for a light adapted eye (and lying at the basis of the adopted system of light quantities and units) with the spectral sensitivity of an eye adapted to darkness ([8]), we get the following table (p. 26).

From the table it follows that to ensure identical blackout conditions against enemy flyers at a distance of 1 km from the objective, the luminosity of the terrestrial crusts for blue lamps should be approximately one-fifth of the value for white ones. A comparison of white light with the yellow light of sodium lamps

Ratio of the Values of the Luminosity of Terrestrial Crusts Illuminated by BB and WB Lamps (blue and white blackout lamps) Giving Identical Blackout Conditions (taking into account the spectral reflection factor of the terrestrial crusts and the scattering of light in the atmosphere) (Observation distance, 1 km)

Terrestrial crusts	E_{BB}/E_{WB}	Terrestrial crusts	E_{BB}/E_{WB}
Fine sand	0.15	Dry greenery . .	0.15
Coarse sand	0.17	Plywood	0.18
Semi-swamp land .	0.16	Snow	0.13
Fresh greenery . . .	0.19		

shows that luminosity under yellow light may be 1.5—3 times higher. For an observation distance of 10 km the luminosity of terrestrial crusts under blue light should be approximately 3.5 times smaller than under white light.

These quantitative relations still do not give us the right to draw any conclusions as to the expediency of employing one or another light source.

3. The choice of the light source depends upon the purpose of the blackout lighting ([16–15]).

Let us examine for each type of blackout lighting the dependence of the visual functions characterizing the visual work of the flyer and the person on the ground upon the wave-length of monochromatic light.

Fig. 1 refers to orientation illumination. The solid curve characterizes the visual work of the flyer and is a curve of equal light sensitivity. The dotted curve

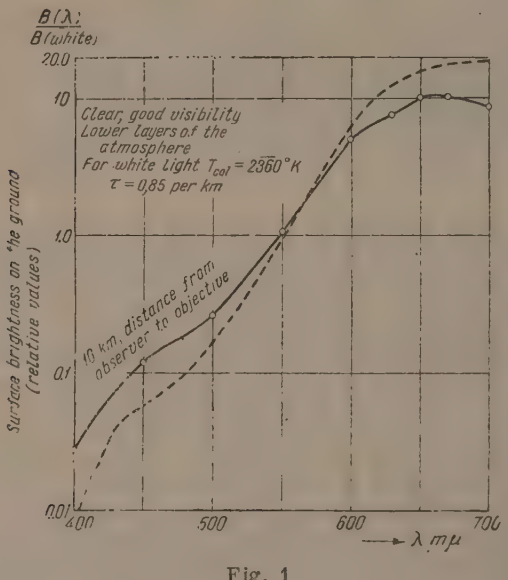

Fig. 1

characterizes the visual work of the person on the ground and is a curve of equal contrast sensitivity. Both curves are plotted in one relative scale of brightness and reduced to one and the same white light ($T_{colour} = 2360$ K). In computing the solid curve the selective transmission of the atmosphere ([16]) was taken into account. On the basis of the experimental data available it may be shown that the course of the curves of equal light and equal contrast sensitivity along the

spectrum is one and the same if the scattering of light by the atmosphere is excluded. In other words, the choice of colour for blackout lighting in a given case is determined not by the diverse visual tasks of the flyer and the person on the ground, but by the selective scattering of light by the atmosphere. That region of the spectrum where the dotted curve passes below the solid curve is advantageous for orientation illumination. This is a sector of the spectrum that begins in the yellow and passes to the blue, where the divergence of the curves is the greatest. Therefore the use of short wave light may be recommended for «orientation illumination».

An entirely different picture is obtained in the case of «working illumination», as is seen from Fig.2, which is plotted in the same manner as the previous one, except that here instead of a curve of equal contrast sensitivity a curve of equal visual acuity has been plotted. The course of the curves in the given case is so different that the selective transmission of the atmosphere can already have no essential influence. Reasoning as previously, we arrive at the conclusion that for «working illumination» it is expedient to employ light beginning approximately with 540 mμ and of longer wave-length. On the same figure have been plotted the sodium lines, showing that the yellow light of sodium lamps is exceptionally advantageous for blackout lighting.

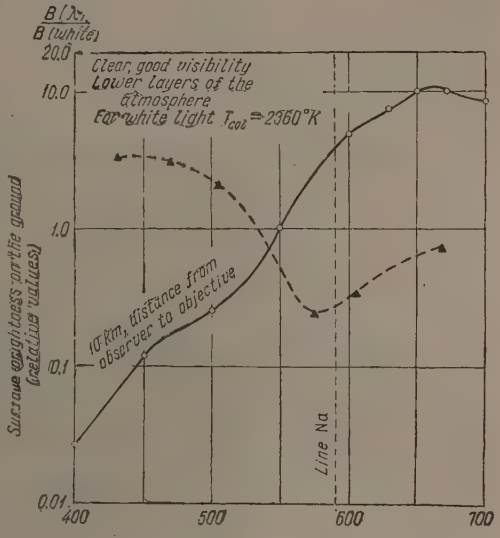

Fig. 2

Passing farther into the red region of the spectrum is of no advantage, since it does not lower visibility from an airplane or improve illumination on the ground. Thus, the middle region of the spectrum right up to yellow-orange colours and, of course, the «white» light of incandescent lamps may be recommended for blackout «working illumination».

The final decision as to what light source to employ for blackout illumination will also depend upon the permissible distortion of the colour of the objects that people on the ground are handling, and also upon the expenditures it involves.

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Received
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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

GEOPHYSICS

THERMAL METHOD OF PROSPECTING *vs.* GRAVITATIONAL AND ELECTRIC METHODS

By S. S. KOVNER and B. L. SHNEERSON

(Communicated by O. J. Schmidt, Member of the Academy, 20. X. 1944)

While exploring the ways in which the results obtained in the study of the thermal condition of rocks could be made serviceable to the purpose of prospecting for minerals ([1–4]), the authors attempted to estimate the thermal field of the section of a region for which the most widely used and reliable geophysical methods of prospecting, that is, the gravitational and the electrometric methods, gave no positive results.

As a first instance, the authors had chosen an area of the Artinskian massif Termen—Elga, sufficiently well explored by deep drilling, and thoroughly prospected by the gravitational and the electric method.

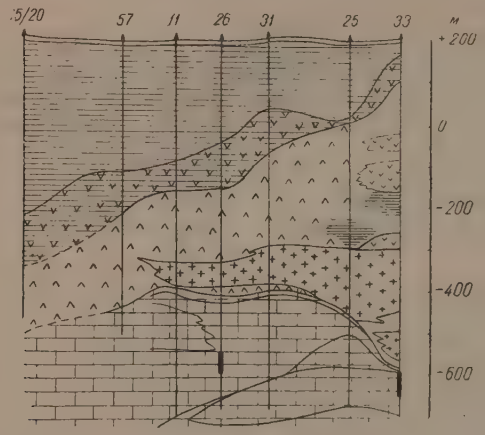

Fig. 1

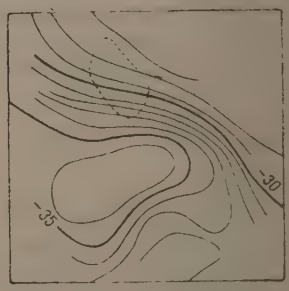

Fig. 2

Fig. 1 shows the section of this massif taken from the map compiled by A. A. Trofimuk in 1941.

It should be borne in mind that from its geological structure the Termen—Elga massif is not an exception. Analogous structures are found in the Ishimbaevo deposits often enough (Kusyapkulovo, Karaganovo, *etc.*). A characteristic peculiarity of the massifs of this kind is that on the one limb of the structure lighter rocks are developed to a considerable extent, as for instance, rock salt, and on the

other limb, heavier rocks predominate, such as anhydrites, both types being of Kungurian age. The Kungurian rocks are overlain by a Ufa series of red rocks.

The electric resistance of the Kungurian rocks is high, approaching infinity, and electrical prospecting practically does not distinguish them. Electrical prospecting, as is known, in such cases records only the bottom of the red beds, and therefore in our example does not give any indication as to the structure of the underlying rocks.

Since the density of limestones, equal to 2.65, is an intermediate one between that of rock salt and that of anhydrite, constituting 2.15 and 2.90, respectively, the effect of the Artinskian massif is generally absorbed by the stronger effect of anhydrite, and is not distinguishable on the gravitational map. An example is presented by Fig. 2, in which the gravitational field of the Termen—Elga region is shown. The figure is a portion of the map of the State Union Geophysical Trust («GSGT») taken from the report of B. L. Shneerson on the Bashkirian Petroleum Expedition of the Academy of Sciences of the USSR in 1942.

As an examination of this map will show, the gravitational minimum, whose origin is due to a great accumulation of rock salt and which attains a value of —36 milligals, is located 1.5—2 km south of the Artinskian massif, shown in the map with dotted lines. The increase in gravity, which owes its origin to a combined action of limestones, anhydrite and red rocks, continues north of the massif: hence from the behaviour of isolines the existence of the massif cannot be revealed on this map.

In this way the electrical and gravitational prospecting whether applied singly or combined, furnish no data for disclosing Artinskian massifs of the type under investigation.

A theoretical investigation of the thermal conditions in the Termen—Elga type was made by S. S. Kovner ([3]). As shown in Fig. 1 of this paper ([3]), the nature of the roof of the Kungurian was taken into consideration and introduced into the calculations, but at the same time the difference between the thermal conductivity of rock salt and anhydrite was neglected.

In the present paper the authors schematize the geological section of the Termen—Elga type in the sense that the roof of the Kungurian is considered to be horizontal, which in the given case is sufficient, but at the same time the difference in the heat-conduction coefficients of all the four rocks is introduced into the calculation, and the configuration of the rocks used is nearer to the actual.

Fig. 3 presents a section, the thermal field of which was calculated: the Artinskian massif, the top of which lies at a depth of 600 m, and the bottom at the depth of 1100 m, has rock salt on one of its limb and anhydrites on the other. The thermal conductivity K, $[K] = \frac{\text{cal.}}{\text{cm. sec. degrees}}$, of the Ishimbaevo rocks is derived from the same sources as in the papers ([3], [4]), and are, respectively:

Artinskian limestone	$K_1 = 0.0047$	Anhydrite	$K_3 = 0.0074$
Rock salt	$K_2 = 0.0068$	Red rock	$K_4 = 0.0035$

The calculation was rather laborious*, since the asymmetry of the figure necessitated a large number ($18 \times 27 = 486$) of squares. So it was carried out by a refined and time-saving method described in ([2]).

To calculate the field of the thermal anomaly the profile shown in Fig. 3 was compared to the profile in which the Artinskian massif is supposed to be absent, and the salt, to be in direct contact with anhydrites along a vertical plane lying at the central section of the massif. As seen from the upper part of Fig. 3, the effect of the thermal anomaly attains at a depth of 100 m from the surface the value of 0.63° C, and at a depth of 200 m, the value of 1.28° C.

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* The calculation was made by Z. S. Ivanova, junior research worker of the Institute of Theoretical Geophysics, to whom the authors' thanks are due.

The theorerical computation consequently shows that thermal measurements carried out at definite depths and accompanied by a parallel computation of the thermal field undisturbed by the presence of the massif, can furnish indica-

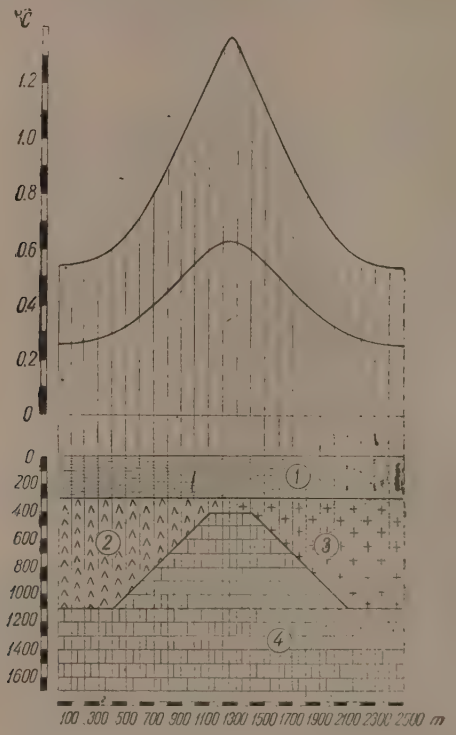

Fig. 3. *1*, Ufa series; *2*, anhydrites; *3*, rock salt; *4*, Artinskian limestone

tions both as to the presence of a buried massif of the considered Termen—Elga type, and as to the place of its occurrence, while the gravitational and the electric methods of prospecting in this case give no indications whatever.

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1945. Volume XLVII, № 1

CHEMISTRY

ON THE TRANSFORMATIONS OF THE MACROMOLECULE OF CELLULOSE IN AN AMMONIACAL SOLUTION OF CUPRIC HYDROXIDE

By O. P. GOLOVA

(Communicated by A. N. Nesmeyanov, Member of the Academy, 13. VII. 1944)

From molecular weight determinations carried out by different physical-chemical methods cellulose invariably appears, and is now generally considered to be a linear molecule containing 2500—3000 glucose units. But the upper limit of the degree of polymerization has not yet been established, and the way the different chains are bound together, the mechanism of addition of non-glucose structural elements and the structure of the chains' ends — none of these questions has so far received a definite answer.

Using the most perfect methods for the purification of nitrogen in the determination of the specific viscosity of cellulose in an ammoniacal solution of cupric hydroxide, the author was able to determine the degree of polymerization of cotton cellulose as 4750 glucose units, the molecular weight of cellulose being thus 720000, the highest molecular weight determined by this method [1].

Values of the same order were obtained for the molecular weight of cellulose in a subsequent work when as the inert medium helium was used instead of the purest hydrogen.

Calculation showed that purification of nitrogen from oxygen up to 0.0001 per cent was nearly equivalent to the use of a vacuum about 10^{-4}. By using both purified nitrogen and a vacuum about 10^{-6}—10^{-7} we were able to bring the content of the oxygen molecule in 100 cm^3 of the reaction space up to a value of $3.57 \cdot 10^5$.

After the elimination of air from the ammoniacal solution of cupric hydroxide and from the histological canals of the cellulose fibre in a specially devised apparatus, the solutions of cellulose in an ammoniacal solution of cupric hydroxide were investigated. For cotton cellulose purified according to Corey and Grey [2] [the method has been described in a previous paper [1]] the degree of polymerization was found to be 9500—10000 glucose units in the chain, which corresponds to a molecular weight equal to 1620000.

Taking into consideration the destruction of cellulose proceeding, as found by us, during purification from pectic substances, waxes and fats, the true value for the molecule weight of native cellulose should be higher than the figure found.

This follows as well from the fact that in an ammoniacal solution of cupric hydroxide, as found experimentally by us, by introducing into the reaction medium $3.57 \cdot 10^{14}$ molecules of oxygen a rapid change in the specific viscosity takes place, this leading to a degree of polymerization equal to 2500—3000 glucose units. When $0.6 \cdot 10^{20}$ molecules of oxygen are introduced into the ammoniacal solution of cupric hydroxide, the degree of polymerization decreases to 50—60 glucose units in the chain.

Oxidized cellulose isolated from the ammoniacal solution of cupric hydroxide, having a degree of polymerization about 50—60 glucose units, was investigated for the length of the chain by the method of the oxidation of final aldehyde group [iodine number method according to Bergman and Machemer ([3])] and by determining the yield of glucuronic acid according to Ivanov and Kaverzneva. The value of 50—60 glucose units was obtained again for the degree of polymerization.

The data obtained on the molecular weight of cellulose make it possible for theoreticians to form new conceptions on the limits of polymerization-condensation processes taking place in nature, and to men of practice it shows the theoretical possibilities for most rational ways of making use of the properties of native cellulose.

The possibility of using the properties of the solutions of cellulose derivatives for determining the degree of polymerization of cellulose lays beyond the scope of this paper.

The methods in which the determination of the molecular weight of cellulose is based on the production of its derivatives depend for their reliability upon whether or not the theory of polymer-analogs is true.

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1945. Volume XLVII, № 1

CHEMISTRY

SOLUBILITY IN THE SYSTEM $CaCO_3 - CaSO_4 - NaCl - CO_2 - H_2O$ AT 25° C

By E. B. SHTERNINA and E. V. FROLOVA

(Communicated by I. I. Chernïaev, Member of the Academy, 22. VI, 1944)

The present work is part of a research carried out under the guidance of Professor A. V. Nikolaev for the Kuibyshev Central Water Power Plant. Its purpose has been to find out how long hydroelectric structures made of limestone and dolomite can resist erosion by river water.

A survey of the literature on the solubility of calcite and gypsum in the presence of sodium chloride and carbon dioxide has been made by Orlov (1) and Frear and Johnston (2).

The calcite (Iceland spar) and gypsum used in this investigation were colourless, transparent specimens from Maidantal in the Kazakh SSR and Karaul-Khana in the Uzbek SSR, respectively.

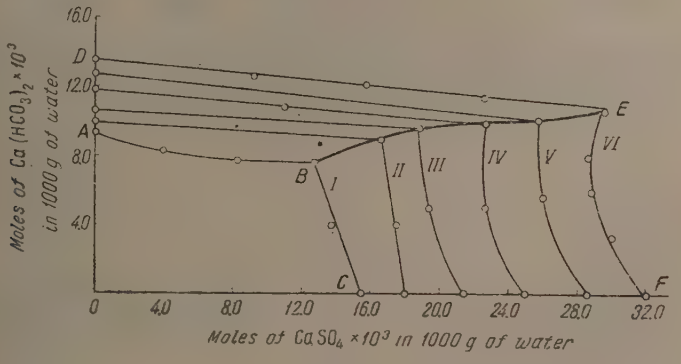

Fig. 1. Solubility in the system $CaCO_3$—$CaSO_4$—NaCl—CO_2—H_2O at $P_{CO_2} \cong 1$ atm. and $T = 25°C$. *I*—0; *II*—0.25; *III*—0.50; *IV*—1.00; *V*—1.50; *VI*—2.00% NaCl

The solubility experiments were carried out in the standard 250 cm³ pyrex vessels used in our institute. The vessels were immersed in a constant temperature water bath kept at 25±0.05° C.

Equilibrium was attained very slowly, on the average after about a month.

The solid phase invariably consisted of calcite and gypsum. The main results of a study of the liquid phase are given in the table and Figs. 1 and 2.

Solid phase	$Ca(HCO_3)_2$		$CaSO_4$		NaCl		Partial pressure of CO_2 (in atm.)	Density d_4^{25}
	g per 1000 g of water	moles× 10^3 per 1000 g of water	g per 1000 g of water	moles× 10^3 per 1000 g of water	g per 1000 g of water	moles× 10^3 per 1000 g of water		
Calcite	1.478	9.12	0	0	0	0	0.9555	0.9989
»	1.621	10.00	0	0	2.500	42.78	0.9639	1.0007
»	1.723	10.63	0	0	5.027	86.01	0.9593	1.0026
»	1.913	11.80	0	0	10.204	174.57	0.9658	1.0065
»	2.073	12.79	0	0	15.000	256.62	0.9645	1.0101
»	2.195	13.54	0	0	20.486	350.48	0.9671	1.0141
Gypsum	0	0	2.091	15.36	0	0	0.9632	0.9993
»	0	0	2.450	18.00	2.500	42.77	0.9750	1.0013
»	0	0	2.916	21.42	5.001	85.58	0.9777	1.0034
»	0	0	3.429	25.19	10.145	173.58	0.9711	1.0077
»	0	0	3.884	28.53	15.002	256.65	0.9750	1.0114
»	0	0	4.356	32.00	20.013	344.41	0.9687	1.0153
Calcite+gypsum	1.258	7.76	1.739	12.78	0	0	0.9726	0.9999
Do	1.475	9.10	2.258	16.59	2.526	43.22	0.9653	1.0023
»	1.580	9.75	2.533	18.61	5.076	86.85	0.9782	1.0044
»	1.614	9.96	3.053	22.43	10.133	173.39	0.9597	1.0084
»	1.652	10.19	3.496	25.68	15.206	260.15	0.9517	1.0124
»	1.736	10.71	4.005	29.42	20.228	346.12	0.9684	1.0165
Calcite	0.129	0.80	0	0	0	0	0.0013	0.9968
Gypsum	0	0	2.096	15.40	0	0	»	0.9989
Calcite+gypsum	0.040	0.25	2.244	16.49	0	0	»	0.9991
Calcite	0.194	1.20	0	0	10.278	175.82	»	1.0042
Gypsum	0	0	3.505	25.75	10.278	175.83	»	1.0074
Calcite+gypsum	0.050	0.31	3.598	26.43	10.285	175.94	»	1.0071
Calcite	0.243	1.50	0	0	20.350	348.11	»	1.0113
Gypsum	0	0	4.414	32.43	20.125	344.28	»	1.0155
Calcite+gypsum	0.061	0.38	4.509	33.12	20.085	343.58	»	1.0151

The equilibrium diagram in Fig. 1 has been obtained at a partial pressure of carbon dioxide close to one atmosphere, and the diagram in Fig. 2. at a partial pressure of carbon dioxide of 0.0013 of an atmosphere.

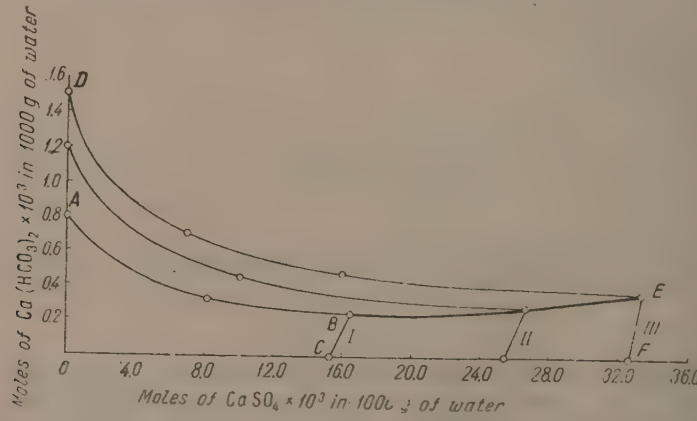

Fig. 2. Solubility in the system $CaCO_3$—$CaSO_4$—NaCl—CO_2—H_2O at $P_{CO_2} \simeq 0.0013$ atm. and $T = 25°C$. *I*—0; *II*—1.00; *III*—2.00% NaCl

The regions *OABC* represent unsaturated solutions of calcite and gypsum in water not containing sodium chloride. The regions *ADEB* include solutions in

equilibrium with solid calcite; *BEFC* are regions of saturation with gypsum. The line *BE* is a curve of the mutual solubility of calcite and gypsum in sodium chloride solutions of different concentrations (from 0 to 2 per cent).

The diagrams make it possible to predict the action of diluted solutions resembling river water in composition on materials containing calcite and gypsum at the given temperature and partial pressures of carbon dioxide. For example, if the composition of some natural water is represented by a point lying in the region *OABC*, such water, being an unsaturated solution with respect to calcite and gypsum, will dissolve these minerals out of the rocks subjected to its influence. Water represented by a point lying in the region *ADEB* will attack only gypsum, but not calcite. The opposite holds true for waters whose composition lies somewhere in the region *BEFC*: such waters will dissolve calcite, but not gypsum.

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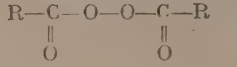
PHYSICAL CHEMISTRY

AN X-RAY STUDY OF THE STRUCTURE OF BENZOYL PEROXIDE

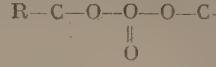
By V. KASSATOCHKIN, S. PERLINA and K. ABLESOVA

(Communicated by V. M. Rodionov, Member of the Academy, 26. IX. 1944)

To explain the properties of acyl peroxides, which behave both as oxidizing and acylating agents, chemists have ascribed them various structural formulae, *e.g.*



(1)



(2)

(3)

Representative of this class of compounds is benzoyl peroxide which, according to Berezovskaya and Semikhatova (1), exhibits unique properties during catalytic decomposition (it is not decomposed by MnO_2 and is decomposed by Pt only in the presence of H_2O).

Discussing the diminution in molar refraction and the shift in absorption to the short-wave region of the spectrum observed by Lederle and Reiche (2) when alkyl radicals had been substituted for the hydrogen in H_2O_2, Berezovskaya and Kurnosova (3) ascribed this effect to electronic stabilization of the peroxide group. And the peculiarities in the Raman spectra of benzoyl peroxide which they themselves have discovered is supposed by them to arise from distortion of the electron system of the peroxide group.

In this connexion they suggest the possibility of a tautometric coexistence of two forms of the molecule: symmetric (1) and asymmetric (2). To our opinion, this interpretation is open to doubt.

The structure of benzoyl peroxide crystals has been studied by Miller (5), Brodie (6) and Groth (4), who assigned them to the holohedrons of the rhombic system with an axis ratio:

$$a : b : c = 0.6287 : 1 : 0.6595$$

The first detailed investigation of the crystal structure of benzoyl peroxide was made by Milono (7) in an X-ray study.

According to this author, the rhombic elementary cell contains two molecules of benzoyl peroxide and has the following dimensions:

$$a = 5.34\,\text{Å}; \quad b = 8.46\,\text{Å}; \quad c = 11.04\,\text{Å}$$

with an axis ratio

$$a : b : c = 0.6313 : 1 : 1.305$$

that is not in agreement with the crystallographic measurements reported by Miller, Brodie and Groth.

The density computed from the X-ray data ($\delta_{calc} = 1.596$) deviates from the value obtained experimentally by X-ray ycnometer method ($\delta_{exp} = 1.503$) by 6.2. per cent.

The author reports the presence of a space group D_2^3 and also describes the probable arrangement of the molecules in the elementary cell.

On the basis of his data he ascribes the molecule the symmetrical structure shown in Fig. 1, which possess a double axis of symmetry. However, Milono's results were not confirmed by us.

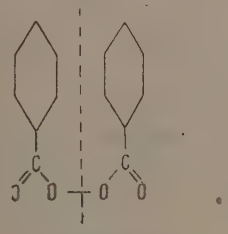


Fig. 1

In the present X-ray study of the crystal structure of benzoyl peroxide, well-formed monocrystals of the compound obtained by recrystallization from ether were employed. To compare the density of the substance and powder photographs specimens obtained by rectystallization from benzene and acetone were likewise examined. The substance was identified by molecular weight measurements (made by the cryoscopic method), its melting point ($T = 103.8^{\circ}$ C) and elementary chemical analysis (carbon content, 69.63 per cent; hydrogen content, 4.46 per cent). Its density, determined by the pycnometer method, was found to be equal to 1.334.

A discrepancy of more than 12 per cent between our density values and Milono's data gave us grounds for questioning the correctness of the structure he propose das well.

Analysis of Laue diffraction patterns and rotation and oscillation diagrams that we obtained with a Miller X-ray spectrograph ($R = 49.27$ mm) and a Siemens' chamber ($R = 77.37$ mm) fixed the symmetry class D_2h and the elementary cell dimensions $a = 8.91$ Å, $b = 9.45$ Å and $c = 14.38$ Å with an axis ratio:

$$a : c : b = 0.622 : 1 : 0.660.$$

Z, the number of molecules in a cell, is equal to 4.

δ_{calc}, the calculated density, is equal to 1.328.

The translation group is P_0.

The reflexes *hkl*, *hko*, *hol*, *okl*, *hoo*, *oko*, *ool* are present in all orders in the X-ray photographs.

From the extinction statistics the space group D_2^1 (P_{nmm}) and the characteristic symmetry of the molecule C_s were found for the reflections of various types $(hkl)(^s)$.

The benzoyl peroxide molecule possesses a plane of symmetry. Starting from the supposition that $L_{C\ldots C} = 3.0$ Å is the shortest distance between the carbon atoms of neighbouring unassociated molecules, that the bond lengths in the molecule are known and that the only possible position is in the form of an elongated molecule along the c-axis (Fig. 2), we found the valence angle $\varphi = 113°15'$ in the peroxide group from the relation:

$$c = 2l_{C-O} \sin\left(\varphi - \frac{\pi}{2}\right) + l_{O-O} + 2l'_{C-C} + 4l_{C-C} + L_{C-C} = 14.33 \text{ Å}$$

Here $l_{C-O} = 1.47$ Å is the length of the C—O bond: $l_{O-O} = 1.45$ Å is the length of the O—O bond in the peroxide group [9]; $l_{C\ C} = 3.54$ Å is the length of the aliphatic C—C bond; $l_{C\ C} = 1.41$ Å is the length of the bond in the benzene ring and $L_{C-C} = 3.0$ Å is the shortest distance between the carbon atom of adjacent molecules along the c-axis.

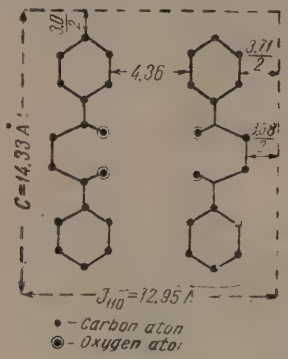


Fig. 2

The space group D^1_{2h} (P_{mmm}) points to the presence in the crystal lattice of molecular groups whose symmetry is characterized by three mutually perpendicular planes. Each group is made up of four associated molecules. The benzoyl peroxide molecule should thus have a plane of symmetry perpendicular to its axis, and should be ascribed a symmetric structural formula (1).

The most probable spatial disposition of the elongated molecules that satisfies the symmetry and dimensions of the elementary cell is shown diagramatically in Fig. 2 (a projection of two molecules in the (110) plane).

If we assume that in the (001) planes, too, the shortest distance between carbon atoms in the benzene rings of adjacent unassociated molecules is equal to 3.0 Å, then the minimum distance between the carbon atoms of the benzene rings belonging to adjacent molecules within the associated group will be found to equal 2.56 Å, while the minimum distance between the peroxide groups is 2.40 Å.

In the crystal lattice the four elongated benzoyl peroxide molecules with their peroxide groups in close contact form an associated group whose centre of gravity lies at the node of a single rhombic space-lattice.

We are now employing this model of the crystal lattice of benzoyl peroxide to obtain more accurate interatomic distances with the aid of the Fourier series.

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PHYSICAL CHEMISTRY

ON THE TEMPERATURE DEPENDENCE OF THE SPECIFIC HEATS OF THE ELEMENTS IN RELATION TO THEIR POSITION IN THE MENDELEYEFF TABLE. I

By N. N. SIROTA

(Communicated by N. P. Chizhevsky, Member of the Academy, 3. VII. 1944)

The temperature dependence of the specific heats of the elements obtained by Debye (1) and the more rigid relation obtained by Born and Karman (2) are in fairly good agreement with the experimental curves for a number of relatively isotropic elements. At the same time experiment has shown that deviation from the Debye curve is the rule for a sizeable number of other elements, mostly substances possessing noticeable anisotropy. For explicity we note that here, as well as in the following discussion, no account is taken of the anomalies in the specific heats near the absolute zero and at high temperatures, anomalies characteristic of the transition metals, the ferro-magnetic elements, *etc.*

Lewis and Gibson (3), investigating the entropy of the elements at standard temperatures in line with the third law of thermodynamics, divided the elements into two classes. Characteristic of the elements in the first class is the satisfactory agreement of the experimental temperature dependence of their specific heats with the Debye curve; characteristic of the elements in the second class is a more or less noticeable deviation from it. Purely empirically it was shown by the authors that a functional dependence corresponding to the Debye expression:

$$c_v = f(T/\theta) \qquad (1)$$

holds for the elements in the first class, and only a modified form of this relation:

$$c_v = f(T^n/\theta^n) \qquad (2)$$

for the elements in the second.

The exponent n is thus a constant characteristic of the given element. Unfortunately, the authors did not give due attention to this side of the problem, having other aims in view.

It is obvious that poor agreement with the Debye curve with respect to the rate of propagation of elastic waves should be expected for bodies with sharply defined anisotropy, inasmuch as in conformity with the initial propositions an isotropic medium was considered. In works based on the Born and Debye method and taking into account the anisotropy of the crystal lattice (4), no general quantitative criterion was given that would permit an appraisal of the influence of anisotropy upon the temperature dependence of the specific heat. Therefore it seemed expedient to obtain at least an approximate quantitative criterion of the influence of anisotropy (dimensional characteristic) of the propagation of the elastic waves in a solid upon the temperature dependence of the specific heat,

similar to what had been done by the author in analysing kinetic curves of phase transformations ([5]).

In the following we confine ourselves to elements, keeping within the framework of the Debye theory, and will not take into account effects of a higher order connected with anharmonic vibrations of atoms, the dependence of the rate of propagation of elastic waves upon their length, *etc.* It is to be assumed that the expressions obtained below may to some extent be extended to compounds as well.

Let us assume that within a parallelepiped corresponding to a grammatom of an element with edges a, b, c, elastic waves are propagated corresponding to the characteristic vibrations of the parallelepiped. Inasmuch as the choice of dimensions of the edges of the parallelepiped is arbitrary, for the sake of clarity in the following discussion let the length ratio of the edges be such that it will be equal to the ratio of the rates of propagation of the elastic waves along these directions. Then, inasmuch as it is a question of the characteristic vibrations of the parallelepiped, it is obvious that a whole number of half-waves fits into each edge:

$$k_{x_{\max}} = \frac{2a}{\lambda_{x_{\min}}}, \quad k_{y_{\max}} = \frac{2b}{\lambda_{y_{\min}}}, \quad k_{z_{\max}} = \frac{2c}{\lambda_{z_{\min}}} \tag{3}$$

Condition (3) and the fact that the total number of possible waves $z_{\max}$ should be equal to three times the number of oscillators $z_{\max} = 3N$, *i. e.* the total number of degrees of freedom of the system, determine, generally speaking, the upper and lower limits of the wave-lengths.

The quantities $k_{x_{\max}}$, $k_{y_{\max}}$ and $k_{z_{\max}}$ referring to corresponding wave numbers, may be regarded as the semi-axes of an ellipsoid $k(x, y, z)$.

Applying Rayleigh's and Jeans' principle, we may determine the total number of positive wave numbers in the space k:

$$z_{\max} = \frac{1}{8} \cdot \frac{4}{3} \pi \left(\frac{2a\ 2b\ 2c}{\lambda_{x_{\min}} \lambda_{y_{\min}} \lambda_{z_{\min}}} - \frac{2a\ 2b\ 2c}{\lambda_{x_{\max}} \lambda_{y_{\max}} \lambda_{z_{\max}}} \right) =$$

$$= \frac{4}{3} \pi V \left(\frac{1}{\lambda_{x_{\min}} \lambda_{y_{\min}} \lambda_{z_{\min}}} - \frac{1}{\lambda_{x_{\max}} \lambda_{y_{\max}} \lambda_{z_{\max}}} \right) \tag{4}$$

where V is the volume of one grammatom.

In reality we are dealing with a discontinuous crystal medium consisting of individual atoms; the oscillatory movement of each of the atoms is described by the superposition of all the waves, regardless of whether they are longitudinal or transverse. It is obvious that the actual vibration does not correspond to any one of them. Thus, the consideration of the propagation of the elastic waves is only a method of the statistical consideration of the question, facilitating its mathematical treatment.

However, the ratio of the probabilities of the displacement of atoms in different directions and hence the ratio of the minimum wave-lengths along these directions is not arbitrary, but is determined by the elastic properties of the medium and the corresponding elasticity moduli, if we keep within the framework of Hook's law, or, in other words, is determined by the anisotropy of the force field of interatomic interaction.

It is convenient to express the ratio of the minimum wave-lengths along the direction of the axes of coordinates through the smallest of them, and the corresponding maximum frequencies of the vibrations of the atoms along the axes of the coordinates $\nu_{x_{\max}}$, $\nu_{y_{\max}}$ and $\nu_{z_{\max}}$ through the maximum frequency ν_z; accordingly, the corresponding components of the rate of propagation of the waves c_x, c_y and c_z will be expressed through the largest of them c_z

$$\lambda_{x_{\min}} = \lambda^{\alpha}_{z_{\min}} = \lambda^{\alpha}_{\min}, \quad \lambda_{y_{\min}} = \lambda^{\beta}_{z_{\min}} = \lambda^{\beta}_{\min} \tag{5}$$

and, in general,

$$\lambda_x = \lambda_z^{\alpha}, \quad \lambda_y = \lambda_z^{\beta} \tag{5a}$$

Making use of relations (5) and (5a) we may rewrite expression (4) in the following form

$$z_{\max} = \frac{4}{3}\pi V\left(\frac{1}{\lambda_{z_{\min}}^{\alpha+\beta+1}} - \frac{1}{\lambda_{z_{\max}}^{\alpha+\beta+1}}\right) = \frac{4}{3}\pi V\left(\frac{1}{\lambda_{z_{\min}}^{q+1}} - \frac{1}{\lambda_{z_{\max}}^{q+1}}\right) \tag{6}$$

where $q = \alpha + \beta$. Or, expressing the wave-length through the frequency and the rate of propagation of the waves $\lambda_{\min} = c/\nu_{\max}$, we shall have

$$z_{\max} = \frac{4}{3}\pi V \frac{1}{c^{q+1}}\left(\nu_{z_{\max}}^{q+1} - \nu_{z_{\min}}^{q+1}\right) \tag{7}$$

Then, on the basis of (5a)

$$dz = \frac{4}{3}\pi (q+1)\frac{V}{c^{q+1}}\nu^q d\nu \tag{8}$$

And from the condition $z = 3N$ we get on the basis of expression (7)

$$\nu_{\max}^{q+1} = \frac{9Nc^{q+1}}{4\pi V}, \quad \nu_{\min} = \frac{1}{2}cV^{-\frac{1}{q+1}} \tag{9}$$

Making use of the Planck distribution, we get an expression for the internal energy

$$u = 3RT\frac{q+1}{x_{\max}^{q+1} - x_{\min}^{q+1}}\int_{x_{\min}}^{x_{\max}} \frac{x^{q+1}}{e^x - 1}dx \tag{10}$$

where, as usual, $x = h\nu/kT$. Long waves play an insignificant rôle in the energy balance and, therefore, without any noticeable error the lower limit may be replaced by zero, at any rate for the case of not too small crystals.

Differentiating the expression for the internal energy with respect to the temperature, we get an expression for the temperature dependence of the specific heat that takes into account the dimensional characteristic of propagation of the elastic waves:

$$c_v = \frac{du}{dT} = 3R\frac{q+1}{x_{\max}^{q+1} - x_{\min}^{q+1}}\left\{\int_{x_{\min}}^{x_{\max}} \frac{x^{q+2}e^x}{(e^x-1)^2}dx - \frac{x_{\max}}{e^{x_{\max}}-1} + \frac{x_{\min}}{e^{x_{\min}}-1}\right\} \tag{11}$$

In the expression obtained q is, consequently, the dimensional characteristic of the propagation of the elastic waves. For $q = 2$ we have three-dimensional distribution; for $q = 1$, two-dimensional; and for $q = 0$, unidimensional. For the unidimensional case expression (11) reduces to the one obtained for a linear chain of atoms ([6]).

The expressions for the linear chain of atoms and for a two-dimensional solid have of late been used by Tarassov (C. R. Acad. Sci. URSS, XLVI, No. 1) in explaining the anomalies observed in the dependence of the specific heat of polymers upon the temperature.

If we limit ourselves to a rough approximation and assume that $c_v = f(T^n/\theta^n)$, as Lewis and Gibson did, it follows from a comparison of expression (11) with the Debye formula that

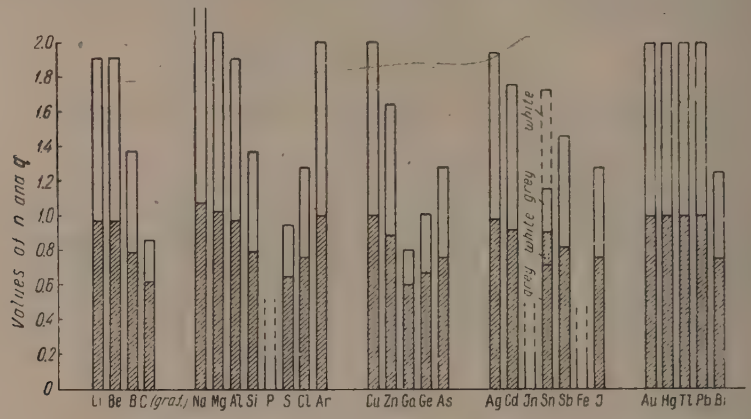
$$q \approx 3n - 1 \tag{12}$$

The exponents q and n are thus connected with the anisotropy of the force field in the crystal lattice, and to a certain measure are its quantitative

characteristic. In other words, q and n are connected with the anisotropy of the energy of the chemical bond within the crystal lattice of an element, conditioned to a considerable measure by its type. On these grounds one may expect a definite dependence of n and q upon the position of the elements in the periodic table. In the annexed figure are given the values of n and q calculated according to Lewis and Gibson and on the basis of expression (12). The data for the computation were taken from the Landolt and Bernstein Tables.

No complete and accurate data being available, the values of n and q given here are of approximate nature.

As can be seen from the figure, a quite definite dependence of these quantities upon the position of the elements in the periodic table actually exists. In most cases for the short series the largest values correspond to the left part of the periodic table. For the elements of the first group, n is close to unity, and q to 2. As



we pass from left to right in the periodic table, n and q decrease noticeably to a certain minimum, which points to a rise in the degree of anisotropy. For the elements of the zero group, n and q again increase. The presence of periodicity in n and q should have been expected on the basis of the conception of the changes in the character of the interatomic bond depending upon the position of the elements in the periodic table. It is in place to point out that here there is a connexion with the Hume—Rothery rule on the change in the coordination number as a function of the number of the group to which the element in the given row of the periodic table belongs.

In conclusion I should like to remark that inasmuch as the space relations in a solid are to a considerable measure conditioned by the energy relations, in the case of anisotropy of the force field the structure of the crystal lattice should be conceived not as the structure of tightly packed spheres, but rather as the structure of tightly packed ellipsoids or geoids.

It seems to us that by pursuing this line notable successes may be achieved.

Received
11.III.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

GEOLOGY

ON A NEW MANGANIFEROUS PROVINCE IN THE SOUTH URALS

By A. G. BETEKHTIN

(Communicated by A. N. Zavaritsky, Member of the Academy, 20. VI. 1944)

Until recently the sedimentary deposits of manganese which are the most widespread in the Urals, have been referred to two different geological epochs, each of these being represented by an independent metallogenic province, *viz.*:

1) The youngest province is associated with Tertiary-Palaeogene siliceous shallow-water sediments in which in the south of the European part of the USSR and in the Transcaucasia are known the largest sedimentary deposits of manganese —the Nikopol and the Chiatury; in the Urals representatives of this province are developed in the northern part of the ridge, along the eastern slope—the Polunochnoye mine, Marsyatsky mine and others.

2) A province of manganiferous jaspers and siliceous shales of Devonian period could also be traced a rather long time ago; it is best developed in the region of Mount Magnitnaya in the South Urals.

After the recent discovery of a number of deposits, the outlines of a new province of sedimentary deposits of Lower Carboniferous period—Tournai-Visé division—became clearly visible, too.

The deposits confined to series of this period are mostly developed in the South Urals. Two belts of Tournai-Visé exposures may roughly be distinguished: the eastern belt, located on the meridian east of the town of Magnitogorsk, and the western one, approximately on the meridian of the town of Asha. Partly exposures of sedimentary rocks of this age with an insignificant area have been discovered not far from the town of Orsk, north-east and south-east of this town. The series to which the sedimentary manganese deposits are confined, consist of siliceous shallow-water sediments (gaizes, spongolites) with limestone bands, not infrequently completely silicified. The deposits are represented both by manganese and iron-manganese ores.

A brief characteristics of the deposits of this province known at present will be given below.

1. North of the Magnitnaya Mount region, among metamorphic rocks of Lower Carboniferous period, two deposits of highly metamorphozed iron-manganese ores are known: (1) the Zmeinaya Gorka deposit (15—20 km south of the Chebarkul station, South-Ural railway), and (2) the Koelga (30 km of the Emanzhelinskaya station).

In either of them the ores occur among siliceous shales and jaspers, subordinate to clay shale beds. The Zmeinaya Gorka deposit is represented by a group of lens-like bodies, steeply dipping westward. The Koelga deposit consists of a number of pockets of irregular shape, and of small stocks among red jaspers. The two deposits had formerly been worked from the surface to meet the demands of the Uralian metallurgical works. The primary ores are represented by manga-

nese silicates (mainly manganiferous garnets and, as it seems, rhodonite), accompanied by magnetite, hematite and other minerals formed in the process of metamorphism, probably of primary sedimentary ores.

2. Kulm deposit which has been known for a long time (110 km south of the town of Magnitogorsk) near the Kulmsky village. According to old data, the ores are confined to siliceous-clay shales of the Lower Carboniferous and occur in the form of thin (up to 2—3 cm) intercalations. In the zone of eluvium over a rather large area pocket-like ore accumulations are encountered. The average content of manganese in the coarse fraction obtained by screening is 33 per cent; in the fine fraction, 27 per cent; in the bolting remains, 28 per cent. It should be noted that this deposit has neither been duly prospected nor studied.

Aside from the deposit near the Kulm village, in this region the following points showing some evidence of manganese ores are also known: (a) in the region of Mount «Belaya Glina», 2—2.5 km south-west of the Kulmsky village; (b) 300—500 m W-N-W of the Novo-Orenburgsky village; (c) 2 km N-E of the Novopotozky village, 100 m north of the road Novopotozky-Novo-Orenburgsky; (d) on the right bank of the Satualda River (not recorded precisely); (e) on the right bank of the Karaganka River (not recorded precisely); (f) near the Briensky village, 6 km south of it, on the left slope of the valley of the Ak-djar River; (g) along the Yanshi-ak-djar River, not far from the point of its discharge into the Suunduk River; (h) 4 km from the Pavlovsky village downstream the Baituk River ;(i) 5 km of the Kvarkeno village, on the road to the Aidyrlya mine; (j) near the Elizavetinsky village, and at other places.

Part of the points named, judging from the geological map of the Urals, is confined to Lower Carboniferous coal-bearing formations (C_1^b). Although this fact does not essentially alter the situation, it is possible, however, that in these coal-bearing formations the manganiferous sediments are still associated with siliceous sediments. Serious attention should anyhow be paid to the numerous indications of the presence of manganese ores in this region. If their association with siliceous shales and limestones of the Lower Carboniferous period be confirmed, all the points listed above should be subjected to preliminary prospecting, and search should be carried on to find new deposits, since the possibility of discovering facies of richer ores is by no means excluded.

3. In the north-western part of the Novo-Orsky village (north-eastward of the town of Orsk) B. N. Nasledov has long ago recorded fragments of massive psilomelane within the deluvium on the Djananka River. No primary deposit has been discovered. According to the data of A. A. Petrenko, rubble of gaizes, spongolites and silicified limestones of Tournai-Visé division are extensively developed in this region.

4. In the Orsk region, 22 km south-west of the town of Orsk, in 1941 a large deposit was discovered—the Akkermanovsky (near the village of the same name). The ($C_1^{v_1}$) horizon, to which the manganese ores are confined, is represented by typical gaizes, siliceous intercalations with spicules of sponges, interbedded with layers of siliceous manganese ores and bands of wholly silicified limestones containing relicts of *Foraminifera* and stems of *Crinoidea*. Locally, at the boundary of the ore-bearing series of intercalations, bands of ferruginous dense siliceous rocks are observed. From data of geological prospecting carried out by E. G. Burova and P. T. Savitsky, the productive series of the Lower Visean stretches over 3 km.

The ore body is of a distinctly bedded form and is represented by a series of comparatively thin (up to 10—20 cm) siliceous ore intercalations of a black colour, alternating with siliceous rocks, containing practically no manganese at all. The total thickness of the ore-bearing series of intercalations attains 5—6 m and is fairly persistent along the strike. The ore consists of crystalline pyrolusite and a considerable amount of macroscopically undistinguishable chalcedony. They are rather sharply bounded from the associated layers of siliceous rocks. As distinguished from the other sedimentary deposits of manganese, the ore intercalations are rather strong through porous masses, which does not support

the possibility of a satisfactory enrichment of these ores by simple methods. At some places there occur rich ores, consisting almost entirely of pure friable pyrolusite.

The grade of ores in the exposed Northern area is not high. As shown by average results of chemical analyses of the samples collected at the Northern area, the manganese content is 18 per cent, the iron content being about 6 per cent, and that of silica, about 50 per cent. A test picking of the ore at the face in the mine (by removing the gangue) showed that in this way an ore mass with a manganese content of 25—28 per cent can be obtained, the silica content being correspondingly lower.

5. In the belt of outcrops of the Tournai-Vise sediments on the western slope of the Urals in the region between the Belaya River in the north and the Sakmara River in the south, the Zianchurin deposit of comparatively poor iron-manganese ores was discovered a few years ago (Bika-Kurgan, Yau-Sankan and Ulu-Elga) near the Chebotarevo village (35—40 km east of the regional centre Isyangulovo and 75—80 km from the Sarantash station, Chkalov railway).

From data of prospecting carried out by K. P. Stolbkov the ore bodies represent bedded deposits extending over hundreds of metres, up to 1—2 m thick, in silicified argillaceous shales, gathered in narrow trough-like folds. From data by M. S. Yankovich who carried out geological surveying in this region in 1941, the ores are confined to the same stratigraphic horizon, traceable in three belts: the one, 5 km; the other, 8 km E-S-E of the Chebotarevo village. The ores occurring in the eluvium consist of small fragments of limestones replaced by manganese hydroxides. They are associated also with siliceous rocks with limestone bands of the Upper Visean.

6. According to A. M. Keller, there is also located a manganese deposit on the Ik River, named the Shigrysh, which has not been explored as yet. Here the manganiferous and siliceous series of sediments of the Visean rests not on the terrigenous Devonian deposits, as it is the case with the Zianchurin deposit, but on Devonian limestones, and transgressively, at that.

To what has been said above it should be added that in the opinion of A. V. Khabakov it would be rational to organize planned quest for deposits of iron-manganese ores in the belt of development of carbonate-ferruginous-siliceous sediments of the Lower Carboniferous period of the Northern Mugodjars.

Thus, from the above survey of the manganese deposits it may be seen that a new province of manganiferous sediments can actually be traced in the Southern Urals. Although we have as yet no considerable deposits of rich ores belonging to this province, the possibility is by no means excluded that such deposits will be discovered, if we consider that no sufficient attention has so far been paid to the study of the problem of the manganese content of the Lower Carboniferous siliceous deposits.

In conclusion it should be noted that the sedimentary manganese deposits of the Lower Carboniferous, as has recently been made clear, are extensively developed within the territory of Central Kazakhstan as well. It can thus be concluded that the Lower Carboniferous time was a definite metallogenic epoch of manganese accumulation in the sediments of shallow-water seas over a vast area, comprising at least the Urals and Kazakhstan.

Received
20. IV. 1944.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

GEOLOGY

ON THE TYPE OF COMPLEX DEPOSITS OF IRON, NICKEL AND COBALT ORES IN THE URALS

By B. P. KROTOV

(Communicated by A. E. Fersman, Member of the Academy, 26. VI. 1944)

The iron ores of the Urals vary greatly as to their composition and quality: along with common ores, there occur such as contain admixtures of manganese, chromium, nickel and cobalt, titanium and vanadium, copper, *etc.*, on the one hand; on the other hand, there occur also particularly pure ores, low in sulphur and phosphorus or, on the contrary, rich in these elements.

Among the iron deposits in the Urals, of particular importance at present are, on the one hand, those of naturally alloyed ores containing chromium, nickel and cobalt; and on the other, of particularly pure ores low in phosphorus. These two types should serve as a basis for an ample supply of the industry with naturally alloyed chromium-nickel cast iron and steel, and especially high-grade metal, smelted on charcoal. Since special interest has been manifested with regard to the naturally alloyed chromium-nickel-cobalt iron ores, work was undertaken with the object to estimate the occurrence of these ores in the Urals.

Up to the present the nickel ores and the iron ores were considered in the Urals ([1]) as two quite distinct and independent types. It was not until recently that among the nickel ores a special variety was distinguished, *viz.* nickel ores rich in iron; and among the iron ores, special types: those alloyed with nickel and chromium, and those rich in cobalt. These types of ores were used each for its special purpose: some for obtaining metallic nickel; others, for producing chromium-nickel cast iron and steel; still others, for obtaining cobalt.

In the course of the work carried on by the author in 1939—40 in the Middle Urals, he became aware of an extensive development of a special type of complex deposits of iron, nickel and cobalt ores, some portions of which, according to their iron, nickel or cobalt content, are practically independent deposits of one or two of these metals. Along with this, in the complex deposits there occur areas where iron ores naturally alloyed with chromium, nickel and cobalt are developed; in these the content of either element taken separately does not attain a workable value which would permit these ores to be considered as payable ores of any one of these metals.

The complex deposits carrying iron, nickel or cobalt ores have not as yet been sufficiently studied, and very few data are available on the content and distribution of iron, nickel and cobalt in them. In the recent years a series of search works has been undertaken by the Institute of Economic Mineralogy and the Ural Geological Service in order to ascertain the presence of nickel in the deposits of iron ores within the area of development of serpentines in the Urals. These investigations were of a purely prospecting and preliminary nature. At that time it had not been safely established that the deposits belong to different

categories and to several geological types, and that the nickel, cobalt and chromium content of all the types vary considerably in the vertical range, and in some of them, in the horizontal, too. Thus, it is not permissible to draw conclusions as to the profitable or non-payable nature of the deposits according to casual samples alone, taken in the superficial portions of the deposits.

Geological investigations and a generalization of the results of prospecting in deposits containing iron, nickel and cobalt ores have shown that these ores occur in deposits of two different categories, and in different parts of the same deposit, or in different deposits of the same ore-bearing area. A part of these deposits belongs to the category of deposits of serpentine weathering. In the vertical, and in some cases also in the horizontal direction in these deposits there is often observed a regular variation in the iron, nickel and cobalt content which may change from a non-paying to an economically profitable one; this is the reason, why originally some deposits or portions of the same deposit were classified as deposits of different metals and are still worked for a definite metal. Only such ores are mined as satisfy the requirements specified for the ores of a given metal, while the remaining bulk of the ores, not satisfying the specifications for other metals, is left unmined or tossed away, no complex utilization being made. On the other hand, it has been now quite definitely ascertained that the ore body usually contains several economic types of ores, which can be spatially delimited from one another because of the regularity established in the variation of the content of the individual components. According to the mining conditions, the individual types (sort) of ores can either be mined separately, or mixed, yielding an ore of an average composition for a given deposit.

These deposits are usually subdivided into two classes: (1) deposits of serpentine weathering, located in the weathering crust of these rocks, and (2) deposits of weathering of the contact-karst type, located at the contact of limestones and serpentines in karst hollows filled with deluvial deposits of the wall-rocks.

The regularities in the distribution of iron, nickel, cobalt and manganese in the two classes of deposits in the vertical direction have been considered in more detail in a previous paper of the author ([2]). In the horizontal direction the content of the alloyed admixtures decreases with distance from the serpentines.

Along with the two classes of complex deposits of iron, nickel and cobalt ores referred to above, in the Middle Urals also developed weathering deposits of the same ores of a mixed type. To this class we refer those deposits in which the distinctive features of the two types are combined. In one part, or in general parts of the area of the deposits, serpentines are exposed, covered with the weathering crust in which the deposit is contained, while at other places of the ore area there are developed limestones with karst or tectonic hollows filled with deluvial deposits containing the ores. In other words, different parts of the same deposit belong to different classes of weathering deposits.

The deposits of the mixed type occur in those places where ultrabasic rocks, later on converted into serpentines, had intruded the limestone formation and formed a number of sills within it. Upon weathering of this formation, karst hollows filled with deluvium were formed in the limestones, while on serpentines a weathering crust was produced. This latter, depending on the climate at the epoch of its formation, belongs to the nontronite or carbonatized and silicified types of the crust.

The concentration of the ore substance in the nontronite crust and in the deluvial accumulations was taking place simultaneously with their formation; the distribution of the metallic compounds in the two parts of the deposit of the mixed type of these classes is quite analogous to their distribution in independent deposits. To deposits of the mixed type belong the Novo-Ivanovskoye, Parushinskoye, partly the Osinovskoye deposit, *etc.*

In addition to deposits of the weathering of serpentines, including the ores named, in the Urals are also found sedimentary deposits of ores of the so-called bauxite type, characterized by a great variety of composition. These deposits

may, on the one hand, resemble in composition those of complex ores of iron, nickel and cobalt of the weathering crust of serpentines, on the other hand, through a series of deposits of intermediate composition, becoming gradually enriched with aluminium and titanium and impoverished in nickel, cobalt, chromium and iron, they pass into pisolitic ferruginous bauxites with traces of alloyed elements.

Such transitions are observed not only within a certain area of development of pisolitic ores, but sometimes also within the whole area of one deposit.

An example of the former case is offered by the transformation of deposits of naturally alloyed ores of the Khalilovo type ([3]), north of the bauxite deposits of the Perevolochinsky type. An example of the latter case is presented by the Rezhev deposits of pisolitic iron-nickel ores, which are replaced by pisolitic bauxites in the east according as the distance increases (*e. g.* the Sirotinsky and Kazenny deposits).

In the past, when bauxites were not yet known in this Country, the deposits of this type were worked for iron ore, and the remaining bulk of the ore was wasted.

From the aforesaid it follows that in the Urals the complex iron, nickel and cobalt ores, on the one hand, pass into iron ores naturally alloyed with chromium and nickel, nickel ores and iron-cobalt ores, and, on the other hand, into bauxites. The diversity of the composition of the ores and the conditions of their formation and application have resulted in that the deposits and the methods of the utilization of ores contained in them have not been sufficiently studied. The papers dealing with iron-nickel and cobalt deposits of the eastern slope of the Middle Urals suggest no strict rules for correlating a given deposit with a definite group. A selective mining of the ores of various sorts from the complex deposits of iron, nickel and cobalt ores will possibly make their working more profitable and rationalize it.

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Received
26.VI.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

MINERALOCY

FORM OF CORUNDUM CRYSTALS AS DEPENDENT UPON CHEMICAL COMPOSITION OF MEDIUM

By K. N. OZEROV

(Communicated by D. S. Beliankin, Member of the Academy, 1. VI. 1944)

Among natural corundum crystals the following four morphological types are rather distinctly differentiated: (1) dipyramidal, (2) long-prismatic, (3) rhombohedral and (4) pinacoidal, according to the prevailing development of a definite face (see table and figure).

A most conspicuous character of the corundum deposits is a strict correlation between the corundum crystals of a definite habit and definite petrographical types of corundum rocks, the latter being always confined to enclosing rocks of a definite petrographical type. This fact can doubtless be accounted for by the genetic peculiarities of corundum deposits alone (see table). But comparatively few deposits of a similar genetic type are characterized by a different form of their crystals. Such are the ruby occurrences in marbles and calciphyres, where corundum is represented by crystals of the II and III types. In one of the deposits mentioned above crystals of a different habit occur jointly: however, each deposit is characterized by some strikingly predominant type of its crystals. Thus, in the Jagdallak deposit (Eastern Afghanistan) long-prismatic crystals are prevalent (II type), while the deposits of Burma furnish mainly rhombohedral crystals of the type III.

An examination of the table will clearly reveal the close relationship existing between the habit of the corundum crystals and the chemical composition of the corundum rocks themselves and the enclosing rocks. Thus, the crystals elongated along the main crystallographic axis, dipyramidal, long-prismatic, as well as those of isometric form—rhombohedral ones, are characteristic only of occurrences in rocks comparatively poor in silica, or containing no silica at all but, on the contrary, rich in bases (Na_2O, K_2O, FeO, MgO and CaO)—alkaline and nepheline syenites, ultrabasic, basic and carbonate. The corundum rocks of these deposits consist of the same minerals that are characteristic also of the enclosing rocks (corundum pegmatites, ruby-bearing marbles), or of minerals undersaturated with silica and rich in bases (plagioclases of the basic series, brittle micas), as well as of those containing no silica at all (spinel, magnetite, hematite, pyrite).

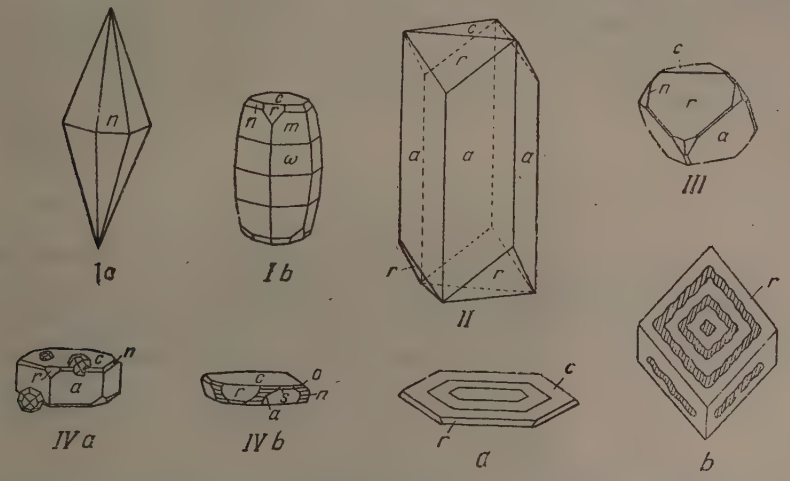
Unlike this, crystals of a pinacoidal habit (IV type)—laminar and tabular—occur almost exclusively in deposits lying among rocks rich in silica: granite-gneisses, gneisses, quartz-mica schists and secondary quartzites (products of metamorphism of acid effusives), occasionally containing very large amounts of excess silica in the form of quartz. The principal and highly characteristic associates of corundum in these deposits are minerals of the composition Al_2SiO_5—andalusite, sillimanite, kyanite and rather frequently quartz, too. The presence of these minerals points that the crystallization of corundum proceeded in a me-

Habit of crystals			Geological peculiarities of deposits					Main deposits
			Corundum rocks			Enclosing rocks		
Type	Prevailing faces	Secondary faces	Name	Main associates of corundum	Characteristic elements	Name	Characteristic elements	
Ia	Pyramids: m(22$\bar{4}$1), n(22$\bar{4}$3), o(22$\bar{4}$5), ω(14.14.$\bar{2}$8.3), *etc.*		Corundum, plagioclasites (kyshtymites, plumasites) and marundites	Plagioclases (An 20—100), margarite, spinel	Ca, Mg	Ultrabasic (peridotites, pyroxenites, serpentines, *etc.*)	Mg, Fe	Borzov (Urals, USSR), Transvaal (Union of South Africa)
			Magnetite-hematitic, chloritoid and margarite emeries	Magnetite, hematite, chloritoid, margarite, pyrite	Fe, Ca	Carbonate (crystalline limestones)	Ca	Near-Irtyash (Urals, USSR), Tawda Ak-Tau (Central Asia, USSR)
Ib		Rhombohedron (10$\bar{1}$1) and pinacoid C (0001)	Corundum, syenite-pegmatites	Potash, feldspars, acid plagioclases, micas (biotite, muscovite)	K, Na, Mg,	Alkaline and nepheline syenites	K, Na	Ilmen Mts. (Ural, USSR), Ontario and Quebec (Canada)
II	Prism a (11$\bar{2}$0) and rhombohedron r (10$\bar{1}$1)	Pinacoid C (0001)	Ruby-bearing marbles and calciphyres	Spinel phlogopite, calcite, dolomite	Ca, Mg	Carbonate (marbles)	Ca, Mg	Eastern Afghanistan
III	Rhombohedron r (10$\bar{1}$1) and pinacoid C (0001)	Prism a (11$\bar{2}$0) and pyramid n (22$\bar{4}$3)						Upper Burma (India)
IVa	Pinacoid C (0001) and prism a (11$\bar{2}$0)	Rhombohedron r (10$\bar{1}$1) and pyramids	Corundum, syenite pegmatites	Potash feldspars, acid plagioclases, micas (biotite, muscovite)	K, Na, Mg Ca	Granite-gneisses	Si	Ilmen Mts. (Urals, USSR), Ontario and Quebec (Canada)
IVb	Pinacoid C (0001)	Rhombohedra r (10$\bar{1}$1), s (02$\bar{2}$1) prism a (11$\bar{2}$0) and pyramids	Spinel-magnetite, feldspar and sillimanite emeries	Spinel, magnetite, hematitic, ilmenite, plagioclases (An 20—50), sillimanite, quartz, garnet	Na, Ca, Mg, Fe, Si	Basic (gabbros and norites)	Mg, Ca, Fe, Si	Sigangoy (Khakassia, USSR), Peekskill (New York, USA)
			Muscovite-, sillimanite- and kyanite- corundum	Sillimanite, kyanite, muscovite	Si, K	Gneisses and crystalline schists	Si	South Yakutia (USSR), Khasi Hills, Assam, Pipra, C. P. (India)
			Muscovite-, andalusite-corundum and secondary quartzites	Andalusite, muscovite, quartz, pyrite, hematite	Si, K	Secondary quartzites (metamorphosed effusives)	Si	Semiz-Bugu (Central Kazakhstan) and Aktash (Central Asia, USSR)

dium containing an excess of silica. In some of these deposits a rather typical associate of corundum is garnet.

An exception to this regularity are deposits of spinel emeries amidst basic rocks, which are xenoliths of contact-metamorphosed aluminous shales incompletely assimilated by the magma. But in this case, too, the acid nature of the shales testifies to an invariable presence in these emeries of a considerable amount of sillimanite.

The above-mentioned relation between the habit of corundum crystals and the chemical composition of the medium in which they have been formed is supported also by the results of numerous and diverse experiments on the synthesis of this mineral ([4]). In these experiments dipyramidal, prismatic or rhombohedral crystals could be obtained only under conditions of sharply basic medium



Habit of crystals: 1) of natural corundum: *Ia*, fusiform; *Ib*, dolioform; *II*, long-prismatic; *III*, rhombohedral; *IVa*, coarsely tabular short-prismatic (with garnet); *IVb*, laminar; 2) electric-synthetic corundum; *a*, laminar; *b*, rhombohedral

rich in such compounds as Na_2O, BaO, CaO, MgO, NH_4 and K_2S, silica being present in insignificant amounts, if at all (experiments by Ebelmen, Fremy et Verneuil, Daubrée, Meunieur, Friedel and Bruhns), while in all other cases the corundum was crystallized in the form of hexagonal laminae. Of particular interest in this connexion are the statements by Morozevich ([3] p. 73) who arrived at the conclusion as to a distinct influence of the chemical composition of the medium upon the habit of the corundum crystals formed. In the experiments of this investigator rhombohedral and dipyramidal crystals were formed in basic anorthite-nepheline magmas only, while «acid magmas (albitic, liparitic) seem to be capable of seggregating corundum as hexagonal laminae only (without rhombohedron and with a very short prism)». It is essential to note that the formation of the latter was invariably accompanied by the segregation of acicular crystals of sillimanite (*i. e.* mullite).

Temperature, pressure and concentration of the solution seem to have no direct influence upon the habit of the corundum crystals formed. The question as to the influence of temperature has more than once been advanced and answered in the affirmative ([1], [2], [5]); however, an analysis of the data available shows that while this agent may be of some import in the formation of corundum crystals, its influence is merely an indirect one (as has, *e. g.*, been noted above for the synthesis of corundum by electric fusion). Experiments show that corundum crystals of a similar laminar habit are produced within a very wide temperature range, from 470 to 2100° C. That pressure is no agent determining

the habit of corundum crystals either is evidenced by the close paragenesis between its invariably laminar crystals and andalusite, sillimanite and kyanite (table, type IVb) which are known to be formed in nature under very different pressures. The same follows from the experiments of Oetling (7) and Noll (6) who obtained at pressures from 125 to 540 atmospheres crystals of corundum of a laminar shape. With sharply different concentrations of solutions even in the same deposits the shape of corundum crystals remains the same, as it is the case, for instance, in deposits among secondary quartzites (Semiz-Bugu), where along with corundum disseminations, amounting only to a few per cent, there are occurrences of nearly monomineral corundum rocks.

It stands to reason that the above-described relation between the habit of corundum crystals and the chemical composition of the medium is to be confirmed experimentally. This refers especially to the nature and mechanism of the physical-chemical processes responsible for the sharp differences in the morphology of crystals of this mineral. The regularity established can already furnish a reliable clue when organizing a rational quest for primary deposits of corundum according to observations on the form of crystals found, *e. g.*, in the blacksands of placers.

Received
1.VI.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

ECOLOGY

A COMPLEX GEOBOTANICAL THEORY OF RIVER ALLUVIAL BOTTOM LAND

By V. P. NOGTEV

(Communicated by B. A. Keller, Member of the Academy, 28. VI. 1944)

The author's complex geobotanical theory of the river alluvial bottom land is based as a basic principle upon the assumed compatibility of phytocoenological and physiogeographical trends in geobotany (1).

The first and chief physical-geographical principle of our theory is that spring flood is considered as a polydisperse and heterodisperse system in the state of turbulent movement.

Polydispersity and heterodispersity of the spring flood are greatly dependent upon its turbulent nature.

Interacting with hydraulic velocities of solid particles in the flood, the vertical components of the velocity of spring flood play the part of «activators» of the spring flood alluviality, aerating at the same time the spring flood.

According to the differentiation of «turbulent forces» of the spring flood, the granulometrical composition of the solid polydisperse phase of the spring flood (its suspensions and sols) is differentiated in the spring flood area, too. But this means that the Tulin (2) ratio $\frac{\text{the 1st group colloids}}{\text{the 2nd group colloids}}$ in freshly precipitated mud sediments is closely and regularly connected with the hydrodynamics of the spring flood and, in particular, with the turbulent structure of the spring flood in both longitudinal and cross-section profiles.

In the polydisperse and heterodisperse systems of turbulent spring flood the processes that take place in the interaction between suspensions, sols and ion solutions are of the following three kinds:

1) Processes of electrolytic coagulation, which lead to the formation of active exchangeable mineral and organo-mineral gels of a high adsorptive power, which are colloids of the 1st group, according to Tulin (2).

2) Processes of reciprocal coagulation of differently charged colloidal micellae of silic acid and humus, on the one hand, and iron and aluminium sesquioxides, on the other hand, which lead to the formation of passive iso-electrical (or with a low negative potential) organo-mineral coagels with a low adsorptive power (the 2nd group colloids, according to Tulin).

3) The «adhesion» processes of the 1st group colloids and the 2nd group colloids which cling to the «framework» of coarse disperse (mainly quartz) particles, suspended during the spring flood; the processes of this kind lead to the formation of organo-mineral micro- and macroaggregates which precipitate on the alluvial bottom land and are fixed there when drying.

Consequently, the adhesion processes are developed on the basis of the preceding processes of electrolytic coagulation and those of reciprocal coagulation.

The ratio $\frac{\text{the 1st group colloids}}{\text{the 2nd group colloids}}$ in micro and macroaggregates of mud sediments will depend upon which of the two processes prevails during spring flood, and upon the above ratio will, in its turn, depend the adsorptive power of mud sediments, as well as their power of cation exchange, *i. e.* the main chemical agents which are of ecological significance for the alluvial bottom land vegetation.

As compared with highly turbulent spring floods, those with weak or weakening turbulent movement are characterized by an increase in the concentration of iron and aluminium sesquioxides, by a decrease in the silicic acid concentration, the ratio SiO_2/R_2O_3 approaching unity, which contributes to the development of the process of reciprocal coagulation and to the formation of heavy inactive clays with a low adsorptive power and a low cation exchangeability.

All this tells upon the stage dynamics of the development of meadow grass-stands in an alluvial bottom land and should be taken into account when working an alluvial bottom land, and preparing it for culture phytocoenoses.

All these processes connected with the characteristic features of the river spring flood as a gigantic natural polydisperse and heterodisperse system in a state of turbulent motion, are of great importance to us; they genetically determine and differentiate the trophics of its habitat in the area of the alluvial bottom land, *i. e.* the «feeding» levels of this habitat, and their «agrophysical» properties (looseness, pore space, compactness, heat capacity, aeration, *etc.*).

The biological principle upon which is based the complex geobotanical theory of the alluvial bottom land is the doctrine of phytosyngenesis stages worked out by the author, *i. e.* of vegetation syngenesis under alluvial bottom land conditions, and of ecological connexions between the stages of phytosyngenesis and the hydrodynamic types of alluvial bottom land.

The forest syngenesis in the alluvial bottom land being out of our consideration, let us point out here the fundamental, or «ascending» stages of the meadow syngenesis we have established:

1. Prosyngenesis, which is the initial stage of vegetation development characterized by vegetation growing in disconnected clumps and consisting partly of ephemeral annuals, with areas of geologically young alluvium not yet overgrown with grasses (vegetation sparingly distributed and growing in clumps). and by a soil-forming process proceeding over disconnected areas (spots) under isolated clumps of vegetation only. The mutual exchange of substances between vegetation and alluvium is also of local disconnected type rather than throughout the area. The competition between the plants is not stable yet, and the vegetation can therefore make but an incomplete use of the productive forces of the habitat.

2. Metasyngenesis is a young coenotically poorly selected stage of development, the vegetation growing in clumps, in close conjunction, with a great number of ephemerals and heterogeneity of grass-stand, the soil and sod formation processes going on all over the area formation. The coenoses still appear in the shape of clumps, they are conglomerative, poorly shaped structure, heterogeneously diffuse, their character being insufficiently pronounced and difficult to diagnosticate.

The bare areas of uninhabited alluvium have nearly disappeared. A mutual exchange of substances between vegetation and alluvium takes place throughout the area (in horizontal plane), being more intense here than in the preceding stage of prosyngenesis. The competition is established almost throughout the area, though the coenoses have but weakly been selected off. The differentiation of relative competing power-activities of different plants which came to confront one another over a given territory, is not clearly pronounced yet, and the grass vegetation is therefore of «mottled» conglomerative character, owing to the clumps of the preceding stage of prosyngenesis.

The sod is not only poorly developed all over the area, but heterogeneous in quality, which agrees with the heterogeneous flora and heterogeneous vegetation at the stage of metasyngenesis. In this connexion the humus accumulative soil

horizon A_1 («accumulation» horizon) is poorly developed and of heterogeneous quality.

The trophic exchangeable soil complex is at the initial stage of development. The stage of poorly developed sod and heterogeneity of grass-stand including ephemerals proceeds on primitive, poorly developed or «immature» alluvial sod-meadow soils.

3. Early anasyngenesis: the sod-rhizome stage of development of better selected vegetative-mobile mesophyl coenoses which rapidly gain ground upon a territory.

Pronounced shifting of the higher gramineae, rhizome leguminous plants and rhizome and root-offspring grass-stand, *i. e.* towards life forms of loose («soft») aerated soils. There is an increase in the combining selection which leads to a greater number of grasses and diffusion of grass-stand at the expense of the decrease in heterogeneous grass-stand, leguminous plants, weeds and conglomerative mottled vegetation. The clumps of vegetation are nearly effaced, the diffuse uniformity and type are sufficiently expressed. A more intense mutual exchange of substances between vegetation and soil leads to a higher intensity of the dark colour of the humus-accumulative horizon A_1, to an increase in its depth and to a greater biological accumulation of ash elements in it, which are extracted from deeper alluvial horizons by root systems.

The trophic exchangeable soil complex is being formed. There is a young dusty alluvial sod-meadow soil without any structure or with loose structure, with a sufficient amount of active colloids (the 1st group of colloids, according to Tulin), with good adsorptive properties and with a great adsorptive capacity.

Examples: *Brometa inermis* and *Agropyreta repentis*; and among forest coenoses, black poplar forests in the alluvial bottom land.

4. Late, or «mature» anasyngenesis: sod short-rhizome and loose-shrubby stage on highly developed (mature) alluvial sod-meadow loamy macrostructural soils.

There is a culmination of competing powers and accordingly of the degrees in the abundance of short-rhizome (meadow foxtail) and loose-shrubby gramineae (timothy, late bluegrass, *etc.*). Mesophytic meadow coenoses (grass-stands) in the alluvial bottom land are of the highest degree of combinative selection. A mutual exchange of substances between meadow mesophytic vegetation and soil, as well as the structural influence upon the soil on the part of meadow vegetation reach ecological optimum to the effect that a high natural trophic regime (a high feeding level) for meadow mesophytes (meadow eutrophic regime) is created. The process of soil sodding is also at its ecological optimum for meadow eumesophytes (agrotechnically «mature») in the sense that sufficient aerobic activity, microbiological activity, nitrate and humus productivity (we mean humus saturated with bases) are retained.

At this stage the trophically exchangeable soil complex gets therefore a further gain and optimal (under the alluvial bottom land conditions) development: (1) in its adsorptive saturated humus part at the expense of «mature» meadow sod, that of foxtail in particular, and (2) in the mineral part, at the expense of optimal silt-clay sediment supplementary feeding on the part of moderately turbulent spring flood.

Meadow vegetation makes the best use of the resources of the most advantageous alluvial bottom land habitats, the genesis of which is physically-geographically connected with the moderately turbulent types of spring flood and moderately active alluviality.

Example: foxtail associations.

5. Telosyngenesis is a stage of transformation of meadow vegetation from «mature» anasyngenetic mesophytic groups into some extreme ecological specialization, namely: (1) into either oxylophytic specialization (meadow peat saturation, (2) or thermo-xerophytic specialization (steppe-like formation), (3) or psychrophytic specialization («cold dryness»), *etc.* The term of «meadow degene-

ration» widely used by agriculturists is applicable to the stage of meadow telosyngenesis.

Therefore the stage of telosyngenesis is a stage of phytosyngenesis of different life forms of extreme ecology. In particular, it is a compact-shrubby stage of meadow.

In the course of this stage a shift of higher competing powers is taking place, and according to them a shift from higher abundance of mesophyte species to representatives of extreme ecological types: such as compact-shrubby oxylophytes (peat-formation plants), compact-shrubby xeromesophytes and xerophytes, psychrophytes («cold xerophytes»), *etc.*

A general physical-geographical cause responsible for this process is the weakening of the alluvial bottom land hydroturbulent state and a pronounced activation of continental zone-climatic factors, *i. e.* the replacement of the turbulent physical-geographical process in the alluvial bottom land by pronounced zonal-continental physical-geographical process. The latter is characterized by the manifestation of its separate elements, such as humidity, dryness, heat, cold, *etc.*, which leads to the shift of the higher competing powers from mesophytes to species of a more extreme ecology.

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Received
28.VI.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

BOTANY

FRATRIAE OF THE PLANT FORMATIONS OF THE USSR AND THEIR PHYLOCOENOGENY

By V. B. SOCHAVA

(Communicated by V. N. Sukachev, Member of the Academy, 4. V. 1944)

The construction of a phylocoenogenetic system of plant formations in this Country should be based on the concept of the divergent development of the phytocoenological categories. This concept follows from the general principles of the Darwinian theory of evolution and is supported by the empirical results of palaeobotany (3). Studies on the structure and florogenesis of certain formations of Western Europe (6), the Caucasus (8) and Far East (9) have proved their partiality relative to the initial Tertiary formations. The Tertiary vegetation which has been more monotonous as compared with the recent vegetation, was represented by various groups distinguished by edificators, by other components and differed in ecological relations. Complete breakdown of the physical-geographical process at the early beginnings of the Quaternary has led to a breakdown of a number of formations. Their characteristic components died out or migrated beyond the boundaries of our floristic provinces, and were only partly absorbed by the newly formed types of phytocoenoses. The latter are the products of a divergent development of a few Tertiary formations which were transformed in different ways and to different degrees. Korzhinsky (4) has been the first to approach the question on the subdivision of the vegetation of this Country into phylocoenogenetic categories; however, his attempt remained incomplete, and his theoretical statements, unformulated.

Among the large taxonomic units of the phylocoenogenetic systematics (9) the fratriae of the plant formations have a determinating significance. They comprise the classes of formations with various dominating biological and vital forms, these classes being derived from a common root, from a morphologically integral Tertiary phytocoenose. The components of a fratria are ecologically isolated but, regarded broadly, the fratriae of the formations are ecologically specialized in a common direction. The fratriae are united to form types of vegetation after the characters of a more remote genetic affinity.

The vegetation of the USSR may be subdivided into the following fratriae of formations (see table).

Fratriae of the Nemoral Type are the most ancient ones. They retain most of the features of the Tertiary vegetation. The first rank in this respect is held by the Manchurian fratria which occupies the primordial territory of Turghay. In the mixed woods of Manchuria which are of a composite phytocoenotic structure (the determinant of the fratria) a stable position is assumed both by the Turghay conifers (*Abies holophylla*, *Pinus korajensis*, *etc.*) and the numerous deciduous trees (*Tilia* species, *Carpinus cordata*, *Juglans manshurica*).

Type of vegetation	Fratriae of plant formations of the USSR
Nemoral	1. Manchurian. 2. Euxine. 3. Hirkanian. 4. Turkestanian. 5. Pontic
Boreal	1. Beringhian. 2. Altay. 3. Angarian
Desert	1. Turanian. 2. Turkmenian. 3. Central-Asiatic
Steppe	1. Daghestanian. 2. Iranian. 3. Black Sea. 4. Mongolo-Dahurian. 5. Kazakhstan
Alpine	1. Kamchatka. 2. Ojrotian. 2. Caucasian. 4. Central Asiatic
Arctic	1. Arctic. 2. Tundra. 3. Mountain-Tundra
Aquatic	1. Palaeohydatophytic. 2. Neohydatophytic

The Euxinian fratria has been formed later than the Manchurian one; however, it has still retained the Tertiary structure rather fully. A strong differentiation is observed in its woods, and monodominant tree-standings are more common. Coniferous standings (*Abies nordmanniana*, *Picea orientalis*) are more sharply delimited from deciduous ones (*Fagus orientalis*, *Castanea sativa*, *etc.*). The Hirkanian fratria, along with the edificators of the Turghay root (*Parrotia persica*, *Fagus orientalis*, *Quercus castaneifolia*, *etc.*), contains ancient South-Tertiary elements (*Diospyros Lotus*, *Albizzia julibrissin*, *etc.*) and autochtonic forms (*e. g.*, *Danaë racemosa*) whose phytocoenological importance is, however, of second order. Many-stage Hirkanian woods of a synovial structure contain no conifers, which is the result of the loss of the one of the characteristics of the Tertiary wood. The Turkestanian fratria is characterized by its simpler structure. The Turghay element had penetrated into the mountains of Central Asia at the latest stages of the development of Tertiary vegetation. Newly formed coenoses had adsorbed the ancient species of Central Asia and finally suffered certain xerophylization. The most typical formations are those of *Juglans regia*, *Malus Sieversii*, *Acer Semenovi*, as well as the meadow groups which are both genetically and morphologically related to the grass-cover of the walnut- and apple-woods. The Pontic fratria (the basic area—wood-steppes and the southern part of the Russian plain, North Caucasus and adjoining regions) was formed in the Quaternary at the expense of the immigrants from the shelters of the Tertiary flora. The historical factor and the physical-geographical regimen determine the monotonous composition of the nemoral vegetation and the simplified structure of the phytocoenoses which had partly adopted the boreal element. Determinant formations are groves of *Quercus robur* and certain groups of the «meadow-steppes».

The Boreal Type of vegetation persists in Asia through the whole Quaternary. Beringhian fratria is the most ancient one, its existence being traced down to the Pliocene. Originally it occupied a considerable area (the Verkhoyansk-Kolyma mountain land and the adjoining regions) where the Angara formations are prevailing now. The edificators of the ancient Beringhian woods in Asia have mostly died out, and only some forms related to them still exist at the American continent. The grassy and shrub vegetation of these formations attained wide distribution in the woods of Siberia and of Europe, being adapted by the formations of the Angara and Altay fratriae. The recent Beringhian formations of the USSR are concentrated in the sub-alpine regions (coenoses of *Betula Ermani*, *Pinus pumila* and some others). The Altay fratria (Altay, Sayany, Western Siberia, Ural, North of Europe) is represented by its primordial formations, which are also of the late Tertiary age; however, its present structure was acquired by it later. In the course of its development this fratria had lost its Mediterranean features, some traces of which still persist even in Siberia (²). The determinant of the fratria is the coniferous wood (*Picea obovata*, *Abies sibirica*, *Pinus sibirica*). Its most ancient structural type (Abiegnum herbosum) had survived in Altay where it is impregnated with the nemoral element. Laricetum herbosum is phylocoenologically related to it (Altay, partly the Urals). The formation of the green-mossy, coniferous wood was followed by the adaptation of grasses and

of small shrubs of the ancient Beringhian tayga. The Angara fratria is the youngest one. Its determinant (the formation of *Larix dahurica*) had arisen in the south of the Siberian shield and in the adjoining zone of the Caledonides. In the middle of the Quaternary there occurred an expansion of the phytocoenoses of this fratria toward the north and north-east, which had disconnected the Altay and the ancient Beringhian taygas from each other.

The Desert Type of vegetation is the product of development of the ancient Mediterranean euxerophytic vegetation, in particular of its autochtonic and palaeogenous elements (7). Some of the components of the desert are of Turghay origin. The Turanian fratria has been formed as a phytocoenological type later than others. Here belong the aggregations of the salt-marsh plants (species of *Anabasis*, of *Salsola*, *etc.*), *Haloxylon* shrubs and associations of absynth (including the regions of half-deserts). The Turkmenian fratria is genetically connected with Iran proper and territorially with the steppes of the Iran fratria in the mountains of the Soviet Central Asia. Decisive importance belongs to the formations of the ephemeroids (*Poa bulbosa*, *Carex pachystylis*) and of the numerous ephemerals. The Central-Asiatic fratria has a fragmentary distribution on our territory (deserts between mountains in the Pamir Altay). Its element is partly adapted by the other fratriae of the deserts.

The Steppe Type of vegetation is historically connected with the desert type. From the phylocoenological point of view it is correct to use the term «steppe» in its strict sense. «Meadow-steppes» belong to the nemoral type; the «dry steppes», to the desert vegetation. The following three fratriae of the steppes were formed at the expense of the Mediterranean element: 1) the Daghestanian fratria, the central position in which assume the «phrygan» phytocoenoses, «garighi» and mountain-xerophytic heterogeneous grass-stands connected with them; 2) the Black Sea fratria (the South of the Russian plain) which was formed in Pleistocene and is now represented by a few formations; 3) the Iranian (mountain) fratria which is ecologically differentiated. It is represented by the *Agropyrum*, *Andropogon* and *Stipa* formations, *etc.*

Some of them (containing *Ferula*, *Prangos*, *Cousinia*) are similar to the savanna, others are characterized by different rhythm and ecology (meadow-steppe of Tyan-Shan, Caucasus). With the Central-Asiatic centre are connected the following fratriae: 1) the Mongolo-Dahuric fratria (Altay, Transbaikalia) with its typical coencsoformers (*Stipa glareosa*, *Aneurolepidium pseudoagropyrum*, *etc.*); 2) the Kazakhstan fratria (the central area are the Kazakh hills, the Turghay plateau). This fratria is characterized by Stipetum Korshinskyi, S. kirghisorum, *etc.*

The steppe type of vegetation has an ancient core; however, fratriae of this type, especially the Black Sea fratria and the Kazakhstan one, are new systems which have not yet fully developed. The coenose-forming steppe plants spread with a considerable speed and easily adapt themselves to changing conditions, so that they penetrate into the formation of other fratriae. In this connexion the formations of different fratriae are mixed together even in limited regions (*e. g.* Ukraine) and thus participate in the landscape to an equal amount. This has been well demonstrated by Lavrenko (5).

The Alpine Type of vegetation has not been yet sufficiently studied, although the problems of florogenesis of highlands had attracted the attention of such outstanding phytogeographers as Engler and Krasnov. Within the limits of large mountain systems the development of the Alpine formations proceeded in an autochtonous way through the contact with the phytocoenoses of the lower belts. Hence the specificity of the formations of the separate mountainous lands and localization within their boundaries of autonomous fratriae. The Caucasian, Ojrotian and Kamchatka fratriae represent the ancient developmental line of the mesophytic phytocoenoses. The Central-Asiatic fratria underwent xerophytization during the Quaternary under the conditions of high mountains. The associations of the Kamchatka fratria (in this Country Kamchatka is the principal centre of distribution) descend to the sea level and come into con-

tact with the boreal phytocoenoses. This fratria has, of course, been influenced by the ancient boreal Beringhian complex.

The Arctic Type of vegetation is the product of degradation, simplification and ecological adaptation to the Arctic conditions of the ancient boreal and partly of the Turghay phytocoenoses proper. Some of the formations had arisen by syngenesis from the Tertiary highland forms (mainly of the Asiatic ones) and emigrants from the boreal centres. Such a syngenesis is typical of the Arctic fratria which is distributed in high latitudes (class of formations of the Arctic undershrub tundras, aggregations of the polar deserts). Many formations of the tundra fratria (bryophytic, lichenic and others) are the result of a reduction of the boreal coenoses. It is necessary to understand Arctic vegetation in a narrow sense so that, for example, dwarf *Betuleta*, *Cladonia* coenoses and some other groups of phytocoenoses are to be included into the boreal type. The mountain-tundra fratria belongs to tree-less rocky summits (goltsy) of East Siberia, to the Urals and the Amur region. In Altay and Kamchatka it is in contact with the formations of the Alpine type.

The problems of phytocoenology and florogenesis of the Aquatic Type of vegetation has not been discussed at any length in this Country. We subdivide this type tentatively into the following two fratriae: 1) the palaeohydatophytic whose basic components are the types of the ancient species (*Trapa*, *Brasenia*, *Euryale*, also *Nymphaea* and *Lemna*) which often are neither phytocoenologically nor florogenetically connected with the recent vegetation of temperate latitudes; and 2) the neohydatophytic whose determinants are species connected with the hydrophytic land-coenoses (*Alismataceae*, *Utricularia*, some of the *Ranunculus* species). The determinants of this fratria are partly facultative hydatophytes which grow also in low marshes.

The division of the plant covering into fratriae as suggested above is a typological one and should not be considered as a regional. The vegetation of a limited district may be composed of formations belonging to different fratriae. There exists nevertheless a geographical localization of the fratriae, which is the result of a differentiation of the recent physical-geographical process and of the palaeogeographical agents. The development of the vegetation is one of the agents of evolution of the physical-geographical complex. This thesis of the modern phytogeography has been developed by Sukachev ([10]) in his theory of the development of phytocoenoses. The effect of radiant energy upon climate, soil formation, lythogenesis and the relief of the surface, which underlies the physical-geographical process ([1]), in the majority of cases occurs not directly but through the vegetation which is often playing the rôle of a sensibilizer. Owing to this, the structural and other characteristics of the phytocoenoses determine the intensity and the external expression of the landscape-forming process in a definite point of the globe's surface.

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Received
4.V.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

PLANT PHYSIOLOGY

CRITICAL PERIODS IN THE NITROGEN NUTRITION OF SUGAR-BEET

By T. T. DEMIDENKO

(Communicated by A. A. Richter, Member of the Academy, 12. VII. 1944)

As critical periods in the mineral nutrition are to be considered such time intervals in the development of plants during which the lack or deficiency of a definite element goes to reduce the yield and impair the economic value of the plant.

The scarcity of information as to the critical periods in the development of sugar-beet made us undertake in 1940 experiments with water-cultures of this plant, in order to correlate the critical periods due to nitrogen and potassium deficiency to the corresponding periods in the development of sugar-beet. The choice of the two elements mentioned above was determined by the fact that they are absorbed by the sugar-beet in large quantities; at the same time both the quantity and quality of the yield of this crop largely depend upon a seasonable supply of nitrogen and potassium.

The experiment was carried out with Beloussov's solution whose composition was modified according as the plants were withheld from nitrogen. In nitrogenless solutions calcium nitrate was substituted for with gypsum. The critical periods were determined either by withholding sugar-beet during various intervals of time from nitrogen, or else by the method of «void windows» (in the experimental scheme the use of the latter method is denoted by «O»), while «H» denotes such cases where the plants were given full nutrient solution.

The results of the yield-records and of the analyses of the chemical composition of the beet are to be found in the table.

Critical Periods of Nitrogen Nutrition in Sugar-Beet

No. of exper. run	Scheme of experiment					Yield in g			Dry weight of root, g	Sugar, %	Content of nutrient elements in dry substance of root		
	Planting	Pairs of roots				Root	Leaf	Total			N	K_2O	P_2O_5
		4	7	10	13								
1	H	H	H	H	H	616	471	1087	129.0	17.0	1.79	2.36	1.19
2	H	H	H	H	O	386	209	595	85.0	17.0	1.42	2.00	1.01
3	H	H	H	O	O	342	250	592	73.0	16.2	1.13	2.32	0.87
4	H	H	O	O	O	154	148	302	34.0	18.4	0.78	1.55	0.98
5	H	O	O	O	O	71	25	96	16.0	16.5	0.65	1.40	0.92
6	H	O	H	H	H	340	250	590	62.0	13.2	1.35	2.68	0.98
7	H	O	O	H	H	202	165	367	40.0	13.7	1.18	2.42	0.95
8	H	O	O	O	H	113	143	256	21.0	14.1	1.87	2.98	1.33
9	H	H	O	O	H	275	258	583	61.0	15.4	1.08	1.95	0.93
10	H	H	O	H	H	324	325	649	65.0	15.1	1.38	2.50	0.89

When the beet was withheld from nitrogen food during 15 days by the time of the appearance of 13 leaf-pairs, the yield of both roots and leaves displayed an abrupt drop which testifies to the nitrogen being continuously absorbed by the plant. Especially conspicuous was the decrease in the yield of leaves, which is accounted for by an intense transveyance of the plastic material to the forming roots. A very sharp negative effect upon the yield was attained by depriving sugar-beet from nitrogen food at the period of the formation of seven leaf-pairs, yet the development of the leaves was not so badly depressed as in the second run.

If the beet is kept on a nitrogenless ration during a 15-day-period corresponding to that of the formation of four pairs of leaves, the yield of both roots and leaves is also found to be reduced considerably. A transfer of the plants to a complete solution till the end of the growing period does by no means prevent the consequences of nitrogen starvation. Thus, in the development of sugar-beet as a critical period with regard to nitrogen nutrition should be considered the time interval during which the plant forms seven pairs of leaves.

The sugar content is not affected by the nitrogen deficiency experienced by the sugar-beet plant during the 15 days that correspond to the appearance of 13 pairs of leaves. The sugar accumulation of the plant is most badly affected if the plant is withheld from nitrogen at the period of the formation of four pairs of leaves, which should accordingly be considered its critical period. Outside this period the plant does not respond to the withdrawal of nitrogen by reducing sugar accumulation in such a conspicuous way.

As may be seen from the Table (runs 3, 7 and 9), the effect of nitrogen deficiency is especially pronounced during 30 days. If the period during which the plants are deprived of nitrogen be extended to 30 days, *i. e.* to a time interval between the appearance of 10 pairs of leaves and the appearance of 13 pairs of leaves, the yield is found to be sharply reduced. However, towards this time the plants have already accumulated a considerable amount of nitrogen, owing to which the yield of root in the experiment was a rather high one, and so was the weight of leaves. The yield of root is still more badly affected if the sugar-beet is withheld from nitrogen during the 30-day-interval between the appearance of seven and ten pairs of leaves; yet the weight of the green mass is less stronglyreduced in that case. There is also a very sharp drop in the yield of such plants as are kept on a nitrogenless diet during the period from the appearance of four pairs and that of seven pairs of leaves. These plants do not recover when transferred to a normal ration: they lag behind the controls considerably, both in the yield of roots and weight of leaf.

Thus, it is evident that the period during which the sugar-beet plant is the most susceptible to the changes in its nitrogen regime (its critical period) is the time-interval between the appearance of 4 pairs of leaves and that of the appearance of 10 pairs; and this is also the critical period with respect to sugar accumulation. Still more pronounced is the influence of nitrogen starvation on sugar-beet plants deprived of this element during 45 days, which may be seen from the table (runs 4 and 8). A withdrawal of nitrogen continued for 45 days, from the appearance of seven leaf-pairs and until lifting is started, has a very strong influence upon the yield of sugar-beet. But it is still more pronounced when imposed upon the plants within the time-interval between the appearance of 4 pairs of leaves and that of 13 pairs. The absorption of nitrogen by the plants within the time-interval after planting and up to the appearance of four leaf pairs has also a great influence upon the yield of this plant. The figures recorded for run 5 point to an ability of young sugar-beet plants to absorb large amounts of nitrogen of which the plant makes good use when deprived of this element after the appearance of four leaf-pairs.

In the course of our investigation of nitrogen nutrition in sugar-beet we were able to trace in young plants the response to nitrogen starvation. The respective figures are given in the records for runs 6, 7 and 8. A withdrawal of nitrogen during 15 days (between the formation of four leaf-pairs and that of seven pairs) results in a sharp decrease in the yield of both roots and leaves, which should

be interpreted to mean that the sugar-beet plant is an important consumer of nitrogen during this period. If nitrogen supply is cut off for a longer period, the yield is badly reduced, and the more so if the period of nitrogen starvation is prolonged to as much as 45 days. From the records for the respective runs it may be inferred that at an early age the sugar-beet plant absorbs large amounts of nitrogen: the cutting-off nitrogen supply for a period as short as 15 days proves to produce a negative quantitative effect upon the yield. The sugar content of beet is also reduced as a result of nitrogen starvation.

The effect of periodical nitrogen starvation imposed upon the plant during a later vegetating period was studied by the authors with the aim to solve the question, at what period nitrogen is used up by the sugar-beet with the best result, and at what stage the plant is storing larger amounts of this element, when young or old. Respective figures are given in the table (runs 2, 3 and 4). When examining the figures illustrating the yield of such sugar-beet plants as had been kept on nitrogen food when young and were withheld from this element at more advanced stages of development, we soon become aware of the fact that insufficient nitrogen supply during earlier age has a more harmful influence upon the yield than total withdrawal of this element during an intense development of the root-system. Insufficient nitrogen supply has a more harmful effect upon the sugar content in younger plants than during the subsequent development stages. This may be accounted for by the fact that at the period of root formation a plant deprived of nitrogen, though developing at a lower rate, is able to use up the nitrogen contained in the organic compounds of old decaying leaves, subjecting them to autholysis.

As to the content of nitrogen, phosphorus and potassium in the root of sugar-beet according to the various experimental runs, it may be concluded that by excluding nitrogen from the nutrient solution, not only the yield, but the uptake of other nutrient elements by the plant were badly affected. As soon as the plants cease to receive full nutrient solution, they begin to absorb within the period of growth less nitrogen, phosphorus and potassium than the control plants. One should, however, refrain from such an uncompromising conclusion when examining the figures illustrating the uptake by the plants of the fundamental nutrient substances (N, P and K) per unit of dry substance synthesized. The consumption by the beet of nutrient elements was found to vary according to the period during which the plant was withheld from nitrogen.

Conclusions. 1. The sugar-beet requires nitrogen throughout the vegetation period.

2. As a period critical with respect to nitrogen nutrition should be considered the time of intense development of the assimilatory apparatus (formation of 4—7 pairs of leaves).

3. The cutting-off nitrogen supply has, irrespective of the development stage, a harmful influence upon the yield both of root and leaf.

4. When supplied at the second half of the growing period of the plant, the effect of nitrogen is to increase the content of colloids in the cell-sap.

5. By excluding nitrogen from the nutrient solution at the period of sugar accumulation (formation of 7—10 leaf pairs) we reduce the quantity of soluble nitrogen.

6. By excluding nitrogen from the nutrient solution the plant's uptake of nitrogen, and also of phosphorus and potassium, is reduced; it has been found that the longer the plants are withheld from nitrogen, the less nutrient elements they absorb, for the deficiency of one of the most important constituents of their food prevents their organism from developing normally.

The Voronezh Regional Experiment Station.

Received 6.VII.1944.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

PLANT PHYSIOLOGY

ECOLOGICAL AND BIOCHEMICAL CHARACTERS OF *BETULA KIRGHIZORUM* SAW.-RYCZG. RESULTING FROM THE PLANT'S ADAPTATION TO SOILS IMPREGNATED WITH SALTS

By I. A. KRUPENIKOV

(Communicated by A. A. Richter, Member of the Academy, 14. IV. 1944)

Under natural conditions of the Naurzum pine-wood and Sypsyn-Agach forest (Seven-Lake region of Kustanay province, Kazakh SSR) the Kirghisian birch (*Betula kirghisorum* Saw.-Ryczg.) is mostly growing on soils with a high content of readily soluble sulphates and chlorides ([1], [2]).

The high salt resistance of the Kirghisian birch is due to the plant's ecological and physiological remodelling that took place in the course of the gradual salt accumulation by the soil-cover at the habitat of the plant. The very origination of this young endemic birch species seems to be related to the progressive alkalinization of humid regions which were at an earlier epoch characterized by *B. pubescens* Ehrh. from which, according to some evidence, *B. kirghisorum* is likely to be derived ([3], [4]). It should be pointed out, however, that in a number of characters *B. kirghisorum* is related to *B. verrucosa* Ehrh. as well, one of the hybrid forms of which it seems to be.

The gradual salt accumulation by the soil of the region inhabited by the Kirghisian birch was connected with the regions being drainless, and also with an intense supply of pulverized salts through the agency of air ([5]).

After having found out that Kirghisian birch was growing on soil rich in salts ([2], [1]) we undertook to investigate, how this species has become adapted to a substratum so high in salts, which is by no means appropriate to trees or shrubs other than halophytes. The Kirghisian birch scarcely shows any such external halomorphic characters as are so conspicuous in *Tamaricaceae*, black saxaul, *Nitraria schoberi*, and other representatives of the halophytic woody fauna. It appeared of interest therefore to trace these ecological and biochemical peculiarities of the plant which have arisen as a result of its newly acquired halophytic habit, or, more strictly speaking, developing anew in the process of the changes taking place in the surrounding medium.

If viewed from the biochemical point of view the Kirghisian birch is found to display highly peculiar features (Table 1). The leaves of this plant are very high in ash elements. Very high is the content of potassium, chlorine and especially of lime (calcium oxide plus CO_2) which the leaves of most woody plants contain in much smaller amounts.

There is evidence available according to which the accumulation of chlorine by plants is accompanied with calcium accumulation ([6]). This is the phenomenon we are facing in the case here discussed. The soils on which the Kirghisian birch is growing contain up to 28 per cent calcium carbonates; up to 51 per cent

Table 1

Leaves of *B. kirghisorum* Analysed for Total Ash Content (in per cent to dry weight of plant) and Chemical Composition of Ash (in per cent to weight of ashes)

Hygrosc. water	Total ash content	In per cent to weight of ashes												
		SiO_2	CO_3	Al_2O_3	Fe_2O_3	MnO	CaO	MgO	K_2O	Na_2O	P_2O_5	SO_3	Cl	Total
7.78	7.99	2.50	17.99	3.48	0.57	0.07	51.14	1.02	11.09	2.44	0.34	3.12	7.12	100.88
7.15	7.80	7.21	10.28	7.35	0.86	0.05	44.56	8.57	6.50	4.48	0.32	3.20	7.18	100.56

calcium oxide is accumulated in its leaves. Accordingly, the Kirghisian birch is a clearly pronounced calcium-loving plant. The intense accumulation of Ca and Cl in the plant's tissues does not result in an increase in the stage of vegeta tion which is observed for plants that do not agree with high calcium contents in the soil ([6]).

Table 2

Data of Chemical Analysis of Water Extract from Leaves of *B. kirghisorum* *

Hygrosc. water	Solid residue	Water soluble organic substances	Alkalinity in HCO_3	Cl'	SO_4''	$Ca^{\cdot\cdot}$	$Mg^{\cdot\cdot}$	$Na^{\cdot}$	$K^{\cdot}$
7.78	17.14	10.28	3.06	0.57	0.25	0.172	0.34	0.39	1.06

* Water extracts were prepared according to the method suggested by M. M. Shukevich ([7]).

If we are to examine the quantity and composition of water-soluble substances contained in the leaves of the Kirghisian birch (Table 2), we become aware again of a number of peculiar biochemical features; *viz.*: an extremely high total content of water-soluble substances, a high alkalinity index, a large amount of water-soluble organic matter against a rather moderate content of mineral substances.

The osmotic pressure of the soil solution prepared from soils under Kirghisian birch does occasionally amount to 30 atmospheres ([2]). Consequently, in order not to perish from the physiological dryness of the soil, the Kirghisian birch must be endowed with a high osmotic pressure of its cellular sap. In the leaves of the plant this condition is realized largely at the expense of accumulation of water-soluble organic matter and, to a much smaller degree, at the expense of readily-soluble mineral salts. Thus, according to its biochemical characters, the Kirghisian birch somewhat approaches the so-called organo-mineral salt-loving plants.

As compared to other birch species, the leaves of the Kirghisian birch are more succulent: their water content attains 80.2 per cent, exceeding the water content in the leaves of *B. verrucosa* by almost 10 per cent (Table 3).

As has already been pointed out above, the Kirghisian birch is characterized by a high concentration of its cellular sap, in which it considerably exceeds *B. verrucosa* (Table 3). It should also be emphasized that even in the case where *B. kirghisorum* is growing on a soil low in salts, it persists rather steadily in its biochemical characters, largely with respect to the accumulation of water-soluble organic matter; consequently, its halophytic habit cannot be considered as a result of individual adaptation of separate plants to soils high in salts.

Table 3

Ecological and Biochemical Characters of *Betula kirghisorum* and *B. verrucosa* Growing in Naurzum Pine-wood (leaves sampled on July 12, 1940)

Birch species	Association	Water in % to weight of live leaves	Per cent to dry matter weight					Osmotic pressure of cellular sap in atm.
			Ash	In water extract				
				organic matters	alkalinity in HCO'_3	Cl'	SO_4''	
B. kirghisorum	Salt-meadow abounding in *Elymus salsuginosus*	80.2	7.99	10.28	3.06	0.57	0.25	24
»	Pine-wood	77.2	5.92	9.13	2.98	0.24	0.17	17
B. verrucosa	Periphery of a meadow slightly impregnated with salt	71.1	5.12	4.07	1.96	0.18	0.15	8
» »	Pine-wood	70.8	4.99	3.91	1.83	0.16	0.13	6

The halophytic nature of this plant is unquestionably a specific and inherent character. The Kirghisian birch when growing on soil practically devoid of salts (pine-wood) would otherwise not preserve its typical biochemical features.

The osmotic pressure of the cellular sap in the leaves of the Kirghisian birch attains 24 atmospheres*, which four times exceeds the figure obtained for *B. verrucosa* (Table 3).

A number of morphological characters of the Kirghisian birch also testify to its adaptation to soils high in salts (a high succulence of leaves, dense pubescence of young shoots, of the tassels, peduncles and even of cuttings).

Conclusions. 1. *Betula kirghisorum* is a typical species thriving on soils impregnated with salts.

2. According to its ecological, biochemical and physiological characters *B. kirghisorum* approaches the organo-mineral salt-loving plants.

3. The creation of a clearly expressed salt-loving plant endowed with a number of typical physiological and biochemical features inherent in halophytes (high osmotic pressure of cellular sap, intense accumulation of water-soluble substances in its tissues, high water content of the leaf) can take place within the range of such a vital form as the deciduous tree.

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Received
12.IV.1944.

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* Determined with the aid of plasmelysis, with a KNO_3 solution; leaves sampled on August 15, 1942.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

PLANT PHYSIOLOGY

STATE OF THE LEAF AND PHOTOSYNTHESIS

By A. A. RICHTER, Member of the Academy, K. T. SUKHORUKOV and L. A. OSTAPENKO

In the previous communications ([1-4]) we have reported the results of our studies of the assimilation power of the leaf, carried out at the Laboratory of Photosynthesis of the Academy of Sciences of the USSR. It has been found that photosynthesis, as measured by the capacity of CO_2 assimilation under constant environmental conditions, attains already at the early stages of leaf development a certain value which in the subsequent remains constant irrespective of the growth and development of the plant. The present report contains the results of the studies of the relations between the assimilation power of the leaf and its state.

Accumulation of the Assimilates in the Leaf and CO_2 Assimilation. It is regarded as rather safely established that the accumulation of the products of photosynthesis leads to a slowing down of the process of further CO_2 assimilation and may even result in a total suppression of this process. Diurnal fluctuations in the intensity of photosynthesis are often considered as being connected with the accumulation of the products of assimilation and with the rate of their translocation from the leaf ([7]). The following experiment was arranged as an attempt to answer the question how soon and to what a degree does an arrest in the translocation of the assimilates tell upon the assimilation power of the leaf.

Leaves of blooming sunflower (*Helianthus annuus*) were girdled on the 9th, 10th and 11th of August; girdling was done by means of a strap of absorbing cotton soaked in chloroform and kept around the petioles until all the cells beneath it were killed. In order to keep the leaves in normal position they were fixed with holders. Throughout the experiment the leaves preserved their normal turgor. Some of them were kept in full darkness with the aid of opaque covers.

Similar treatment was applied to the leaves of sunflower which had been deprived of the flowering heads («decapitated») at the time of their formation. After 39, 66 and 95 hours the accumulation of the assimilates by girdled leaves and their transport from the leaves kept in darkness was recorded by the change in dry weight calculated for the leaf area. Parallel determinations of the CO_2 assimilation under constant conditions were made as before. The results of the determinations are presented in Tables 1 and 2.

It may be seen that decapitation had caused noticeable deviations from the normal physiological condition of the leaf. In the normal plants the accumulation of assimilates has slowed down the synthesis of organic matter (down to 50 per cent within the period from 66 to 95 hours); unlike this in decapitated plants the accumulation of the assimilates has caused an increase in the synthesis in the leaf (up to 875 per cent of the initial during the same period of time). The translocations also differed in the two cases. In the normal plants, as one

Table 1

Accumulation and Translocation of Assimilates in *Helianthus annuus* in Girdled Leaves and in Darkness

Plants		0—39 hrs			39—66 hrs			66—95 hrs		
	Weight of 100 cm² initial area in gs	Weight of 100 cm² area in gs	Gain per 1 hour in dcmgs	Loss per 1 hour in dcmgs	Weight of 100 cm² area in gs	Gain per 1 hour in dcmgs	Loss per 1 hour in dcmgs	Weight of 100 cm² area in gs	Gain per 1 hour in dcmgs	Loss per 1 hour in dcmgs
Flowering	0.4771	0.5225	12 100%	26 100%	0.5480	9 75%	25 96%	0.5658	6 50%	8 30%
Not flowering (decapitated)	0.6651	0.6794	4 100%	11 100%	0.7572	29 725%	23 209%	0.8609	35 875%	35 318%

Table 2

Photosynthesis and Respiration in Girdled and Darkened Leaves of *Helianthus annuus* (mgs CO_2/hour dcmg²)

Plants	Leaf	Photosynthesis at the beginning	39 hrs		66 hrs		95 hrs	
			Photosynthesis	Respiration	Photosynthesis	Respiration	Photosynthesis	Respiration
Flowering	Girdled	11.05	10.36	3.02	9.56	3.27	9.58	2.96
	Covered	»	7.78	1.77	7.96	3.02	8.29	2.52
Not flowering (decapitated)	Girdled	11.10	12.43	5.31	12.99	6.55	10.57	4.36
	Covered	»	8.97	2.98	8.41	2.31	8.36	2.06

should have expected, they dropped in the dark to 30 per cent from the initial value within a period from 66 to 95 hours. In decapitated plants they rised attaining within a 66—95 hours' stay in darkness 318 per cent of the initial value.

In breaking the integrity of the plant (removal of flowers) considerable changes take place in the ability of the leaf to accumulate the assimilates and to translocate them to other organs. In the case here described decapitation made the leaf more plastic. It can be noted that the accumulation and translocation of the assimilates within one hour were on the average (within an interval 39—66 hours) quantitatively similar. The assimilation capacity of the leaf when the assimilates were retained in it varied but slightly in the two series of plants, while the translocation of the assimilates caused a pronounced reduction when the leaves were kept in the dark for a long period. It is difficult to decide for the time being what was the cause of the inhibition of CO_2 assimilation—a long stay in darkness, or a translocation of the materials essential for the photosynthetic activity of the plastids.

Age of the Leaf and Assimilation Capacity. As previously shown (4), the leaf already at its early developmental stages displays an assimilation capacity which remains stable for a long time during its further development. It still remains rather obscure what is the influence of the structure of the leaf upon the physiology of its assimilation apparatus. Willstätter and Stoll (5) had studied leaves of different age showing clear characteristics of age differences as to colour and position on the shoot; both in young and old leaves there was observed a reduced CO_2 assimilation as compared with fully developed leaves of middle age.

In Table 3 are represented the results of our determinations of the assimilation capacity at two ages, *viz.* at full development and at the period of the first appearance of externally tangible signs of senescence, as shown in the colour.

Table 3

Age of Leaf and Assimilation Capacity

Plant species	Young leaf			Old leaf		
	Position of the leaf upon the stalk	Photo-synthesis	Respira-tion	Position of the leaf upon the stalk	Photo-synthesis	Respira-tion
Helianthus annuus	28	11.88	4.45	9	8.51	1.89
Cucumis sativus	13	12.70	4.03	4	11.70	2.43
Vicia faba	10	8.35	1.95	4	5.92	1.68
Zea Mays	10	14.60	3.38	4	13.60	3.48
Nicotiana rustica	10	11.70	4.31	2	4.92	1.40
Beta vulgaris	14	8.91	2.43	3	7.76	2.09
Solanum lycopersicum . . .	12	9.42	3.12	6	7.76	1.70
Lactuca sativa	18	11.09	2.13	6	8.86	1.43
Cucurbita Pepo	30	13.95	5.98	11	9.45	2.33

The assimilation capacity of the leaf decreases with the appearance of the signs of senescence.

Anthocyanine Content and Photosynthesis. In order to bring out the correlation between anthocyanine content and CO_2 assimilation experiments were made with leaves of anthocyanine containing varieties of table-beet and such as do not contain it, and with leaves of *Perilla* sp.; these leaves were taken at different developmental stages. In *Perilla* the anthocyanine content noticeably increased toward the time of blooming. The results of determinations are given in Table 4.

Table 4

Photosynthesis and Anthocyanine Content

Plant species	Leaf colour	Photo-synthesis	Respira-tion
Beta vulgaris	Normal green	8.79	1.33
	Dark red	8.29	1.72
Perilla sp.	Red-green	6.77	1.34
	Dark red	6.03	1.15

The above figures make one agree with Willstätter and Stoll ([5]) that anthocyanine content does not influence directly the assimilatory capacity of the leaf.

Assimilation Power of the Leaf in the Course of the Day. Many studies of the diurnal course of photosynthesis, mainly under natural conditions of growth, are available ([6]). The results obtained testify to a complex relation between photosynthesis and the influence of the various environmental factors upon the plant. It remains obscure whether there is a diurnal periodicity of photosynthesis due to the properties of the assimilating plant or to its assimilative apparatus. Kostychev ([3]) ascribed to the studies of the periodicity a great value, considering the problem of high importance for the development of the theory of photosynthesis.

In Table 5 are presented the results of our determinations of the assimilation capacity of the leaf in the course of the day in sunflower, tobacco and corn.

Disregarding some insignificant fluctuations caused by experimental errors it can be stated that the assimilation capacity of the leaf remains constant in the course of the day.

Table 5

Assimilation Capacity of the Leaf in the Course of the Day

Date	Hour of sampling	*Helianthus annuus*		*Zea Mays*		*Nicotiana rustica*	
		Photo-synthesis	Respira-tion	Photo-synthesis	Respira-tion	Photo-synthesis	Respira-tion
24. VIII	7	9.95	2.36	6.69	1.44	8.93	2.20
25. VIII	9	12.66	4.23	9.05	2.15	9.58	2.63
24. VIII	10	11.40	3.51	8.15	2.25	10.65	2.50
23. VIII	12	9.20	3.61	8.15	1.61	9.83	2.30
24. VIII	13	9.45	3.58	6.91	2.87	11.23	3.71
25. VIII	14	11.75	4.21	8.34	1.57	11.46	3.49
23. VIII	16	12.56	3.42	7.53	1·69	10.28	2.30
24. VIII	17	10.80	2.93	8.67	2.26	10.38	2.71
25. VIII	19	11.65	3.68	7.78	1.68	9.33	1.83

Only some of the agents seem to affect the assimilation capacity of the leaf. Among those which have been analysed above only long shading and externally visible senescence have produced a clear effect.

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Received
9.XI.1944.

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Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1945. Volume XLVII, № 1

ENDOCRINOLOGY

SEXUAL CYCLE IN *SCIURUS VULGARIS* L. STIMULATED BY MEANS OF LIGHT

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(Communicated by L. A. Orbeli, Member of the Academy, 22. IV. 1944)

The phenomenon of the dependence of gonadic activity on the light factor has been experimentally demonstrated for the first time by Rowan ([7], [8]) on *Junco hyemalis*. The author provoked in these birds during the autumn-hibernal period a development of their sex glands to the degree which is reached under natural conditions in spring, by means of a gradual increase of artificial lighting. Rowan's effect was fully corroborated in the subsequent experiments by Bissonnette and others on *Sturnus vulgaris*, which simultaneously illustrated some peculiar details of this dependence of the sexual function on light ([4],[5]). The works of numerous authors and especially those by Benoit ([2], [3]) on ducks studied in great detail the mechanism of the action of light upon the sex glands and the part played in this mechanism by the anterior lobe of the pituitary body, as an intermediate link.

The result of a great number of studies was to prove that in the realization of the seasonal sexual cycle of the majority of bird species a decisive part belongs to light. The gradual increase in the exposure of birds to light during the spring period in Nature causes an increase in the gonadotropic activity of the pituitary body and through it a stimulation of the gonads. An artificial imitation of the spring light regime at another season leads regularly to an activation of the gonads of the animals under experiment. An activation of the sexual cycle is induced even under changed conditions, *i. e.* when the increase of the daily exposure to light proceeds at a higher rate than it actually does in Nature. Contrariwise an artificial decrease in the duration of daily illumination beyond the limits of a certain minimum value or a total exclusion of the factor of light leads to regressive changes in the gonads, characteristic of the quiescent state. Under such conditions the active phase of the sexual cycle does not appear in the course of the whole year.

The dependence of the function of sex glands in mammals on light has not been studied as yet in such detail as in the case of birds, but there can be no doubt as to the identity of the nature of this dependence in both classes of animals. Experiments on field-mice (*Microtus agrestis*, *Peromyscus leucopus*) have demonstrated the possibility of modifying the dates of the sexual cycle in these animals by changing the light regime ([1], [9]). By increasing experimentally the duration of their exposure to light the procreating ability has been successfully activized in the autumn in female and male ferrets (*Putorius vulgaris*) and racoons (*Procyon lotor*). The progeny obtained from the experimental animals was normal ([6]). At the same time it was established by experiments on other mammals (rabbits, marmots, hedgehogs) that modification of the light regime remains without effect on the phase change of the natural sexual cycle in these animals. Whether this is to be explained by the circumstance that under natural conditions the

cyclical character of sexual activity in these animal forms is actually not controlled by light, or the conditions of the experiments were such that they were unable to reveal a dependence which exists in spite of this,—is as yet difficult to decide. At any rate we may regard the dependence of the seasonal activity of the gonads and of propagation on the gradual increase of the exposure to light and on the total duration of daily illumination as established in the case of many monocyclic animals.

It is well-known that many animals fail to propagate in captivity. The usual explanation is that it is scarcely possible if at all to create under artificial conditions the complex of agents necessary for the appearance of the active phase in the function of the gonads and of propagation. In connexion with what has been said earlier it seems possible to assume that light should be regarded as one of these agents in the first place, and this agent is most often overlooked in the practical conditions under which wild animals are kept in captivity. Variations in the duration of daily exposure to light, in its increase or decrease, are, as a rule, a matter of chance, and this cannot remain without effect on the function of the gonads. The creation of such conditions as would approach the natural light regime seems to be the decisive link in the chain of agents which in their totality may create the possibility of propagation of animals in captivity.

The correctness of this assumption could be tested experimentally, especially on animals which had already lived in captivity for a long period and had provisionally lost their propagating power. It was decided to arrange the respective experiments on four squirrels *Sciurus vulgaris* L. (two females and two males) which we had at our disposal and which did not breed in captivity, since they had been captured two years before.

The animals lived in pairs in large cages, surrounded by a fine iron wire-net ($150 \times 150 \times 120$ cm) which stood in a well lighted room. Tree-hollows and pine branches were placed in the cages. Besides grain and cones the squirrels were given carrots, milk and occasionally nuts. The animals had been kept under similar conditions also before the beginning of the experiment. Above each cage at a distance of 60 cm was suspended a 100-watt electric bulb with a white reflector. The experiment was started on December 3, 1940. Every day at 4 p. m. the light was switched on and after an interval of time, increasing day after day, it was switched out. The duration of this additional exposure to light was increased by 20 minutes every day and in a month it reached 10 hours. This maximum exposure was maintained during the following 25 days, *i. e.* till the end of January. After this the additional lighting was cut off.

Between the 10th — 20th of January the animals under experiment showed the first signs of sexual excitation, and a little later coitus ensued. By the middle of February one of the females had a litter of two, and four days later the second female in the other cage had three young squirrels. The progeny of both squirrels remained under our observation for six months. The young animals developed normally. Thus, under conditions of an experimental increase of exposure to light during a period when the total duration of daily lighting (natural and supplementary) equalized 18—19 hours, the active phase of the sexual cycle ending in coitus and normal pregnancy was provoked in squirrels which would not breed earlier.

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Received
15.III.1944.

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